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EXPONENTIALITY VERSUS GENERALIZED PARETO — A RESISTANT AND ROBUST TEST

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Abstract:

- Using resistant and robust methods we propose the statistic $T_n = (F_U - M)/(M - F_L)$ for testing exponentiality versus generalized Pareto, where F_U, F_L and M are, respectively, the upper and lower fourths and the median of a random sample of size n. The statistic T_n is based on the statistic $V_n = (X_{n:n} - M)/(M - X_{1:n})$ used by Gomes (1982) to discriminate extremal models in a similar context but with a higher breakdown point.
  The simulated power of T_n is compared with the simulated power of $U_n = X_{n:n}/M$ and V_n, which can also be used to test the exponential behaviour of the sample data. Although we observe that the power of T_n is lower than the power of U_n and V_n, we show that the performance of the first test is better than the performance of the two other tests when compared to broadened situations and mixtures commonly used to evaluate resistance and robustness.

Key-Words:

- *generalized Pareto distribution; breakdown point; resistance; robustness; broadened situations; mixtures.*

1. INTRODUCTION

Given the importance of the generalized Pareto distribution in Statistics (*e.g.*, analysis of POT data) we propose a test for testing exponentiality versus generalized Pareto which, although it is not the best one among possible tests, is however resistant to disturbing data, and robust in the sense that it is not sensitive to some departures of the assumptions inherent to a chosen probabilistic model.

Some statistics have been proposed to test the exponential behaviour of samples from generalized Pareto populations, specially for the von Mises–Jenkinson parametrization of the distribution, *i.e.*,

$$F_\beta(x) = 1 - \left(1 + \beta\,\frac{x}{\delta}\right)^{-1/\beta}, \qquad 1 + \beta\,\frac{x}{\delta} > 0, \quad x > 0\ ,$$

where $-\infty < \beta < \infty$ is a shape parameter and $\delta > 0$ a scale parameter. One test that can be used is based on the statistic $U_n = \frac{X_{n:n}}{M}$ (Gomes and van Monfort, 1987). Another possible test is based on $V_n = \frac{X_{n:n}-M}{M-X_{1:n}}$ which was used by Gomes (1982) to discriminate extremal models in a similar context. However, since U_n and V_n are both functions of extreme order statistics, they possess a disadvantage, a zero breakdown point, in the sense of Hampel, as defined below:

Definition 1.1. A statistic T has an α breakdown point $(0 \le \alpha \le 1)$ if the proportion of the sample data that can be replaced by arbitrarily other data with T remaining bounded approaches α.

As an alternative to the tests mentioned above we propose the test statistic

$$T_n = \frac{F_U - M}{M - F_L}\ ,$$

where F_U and F_L denote the upper and lower fourths and M the median of a random sample of size n, with a higher breakdown point (approximately equal to 0.25).

In section 2 we obtain the sample distribution of T_n under the null hypothesis $\beta = 0$ (*i.e.*, exponential behaviour) as well as the limiting distribution. In section 3 the power of the tests T_n, U_n and V_n are compared and the performance of each one is evaluated under broadened situations and mixtures in order to determine their resistance and robustness qualities.

2. SAMPLE DISTRIBUTION OF T_n UNDER THE HYPOTHESIS OF AN EXPONENTIAL PARENT

Let $(X_1, ..., X_n)$ be a random sample from an exponential distribution with distribution function

$$F_0(x) = \left(1 - e^{-x/\delta}\right) I_{]0,+\infty[} \ ,$$

$(\delta > 0)$ and let $(X_{1:n}, ..., X_{n:n})$ be the vector of ascending order statistics associated with the sample.

In order to preserve the ranking symmetry of the fourths from the extremes of the sample, we use the following definition for the $(100p)^{\text{th}}$ sample percentile

$$\xi_p = \begin{cases} X_{\{np\}:n} & \text{if } \ p < 0.5 \,, \\ X_{n-\{n(1-p)\}+1:n} & \text{if } \ p > 0.5 \ , \end{cases}$$

where $\{a\}$ denotes the number a rounded to the nearest integer in the usual way (cf. Casella and Berger, 2002).

Therefore, when n is odd

$$T_n = \frac{X_{n-\{\frac{n}{4}\}+1:n} - X_{\frac{n+1}{2}:n}}{X_{\frac{n+1}{2}:n} - X_{\{\frac{n}{4}\}:n}} \ ,$$

and when n is even

$$T_n = \frac{X_{n-\{\frac{n}{4}\}+1:n} - \frac{1}{2}\left(X_{\frac{n}{2}:n} + X_{\frac{n}{2}+1:n}\right)}{\frac{1}{2}\left(X_{\frac{n}{2}:n} + X_{\frac{n}{2}+1:n}\right) - X_{\{\frac{n}{4}\}:n}} \ .$$

The independence of the spacings of the exponential model yields the independence of the generalized spacings $X_{n-\{\frac{n}{4}\}+1:n} - X_{\frac{n+1}{2}:n}$ and $X_{\frac{n+1}{2}:n} - X_{\{\frac{n}{4}\}:n}$ when n is odd, and therefore the probability density function of T_n was obtained using standard techniques in this case.

When n is even we no longer have independence between numerator and denominator of T_n, and hence the expression that defines the probability density function was obtained calculating the marginal distribution of T_n from the joint probability distribution of the random vector $(X_{\{\frac{n}{4}\}:n}, X_{n/2:n} - X_{\{\frac{n}{4}\}:n}, X_{n/2+1:n} - X_{n/2:n}, T_n)$.

Consequently, if n is odd, the density function is defined by

$$f(t) = \frac{\left(n - \left\{\frac{n}{4}\right\}\right)!}{\left(\left\{\frac{n}{4}\right\} - 1\right)! \left(\frac{n-1}{2} - \left\{\frac{n}{4}\right\}\right)!^2}$$

$$\times \sum_{i=0}^{\frac{n-1}{2} - \left\{\frac{n}{4}\right\}} \binom{\frac{n-1}{2} - \left\{\frac{n}{4}\right\}}{i} (-1)^i \, B\left(\frac{n+1}{2} + \left(i + \left\{\frac{n}{4}\right\}\right) t \, , \, \frac{n+1}{2} - \left\{\frac{n}{4}\right\}\right)$$

$$\times \sum_{j=1}^{\frac{n+1}{2} - \left\{\frac{n}{4}\right\}} \frac{1}{n - \left\{\frac{n}{4}\right\} + 1 - j + \left(i + \left\{\frac{n}{4}\right\}\right) t} \, , \qquad t > 0 \, ,$$

where $B(\cdot\,,\cdot)$ represents the beta function; and, if n is even,

$$f(t) = \frac{\left(n - \left\{\frac{n}{4}\right\}\right)!}{\left(\left\{\frac{n}{4}\right\} - 1\right)! \left(\frac{n}{2} - \left\{\frac{n}{4}\right\} - 1\right)!^2}$$

$$\times \sum_{i,\,j=0}^{\frac{n}{2} - \left\{\frac{n}{4}\right\} - 1} \binom{\frac{n}{2} - \left\{\frac{n}{4}\right\} - 1}{i} \binom{\frac{n}{2} - \left\{\frac{n}{4}\right\} - 1}{j}$$

$$\times (-1)^{i+j} \, \frac{2\left(n + 2j + 2\right) t \; + \; \left(n + 2i - 2j - 2 + 2\left\{\frac{n}{4}\right\}\right) t^2}{\left[\frac{n}{2} + j + 1 + \left(i + \left\{\frac{n}{4}\right\}\right) t\right]^2 \left[\frac{n}{2} + j + 1 + \left(\frac{n}{2} - j - 1\right) t\right]^2}$$

if $0 < t < 1$, and

$$f(t) = \frac{\left(n - \left\{\frac{n}{4}\right\}\right)!}{\left(\left\{\frac{n}{4}\right\} - 1\right)! \left(\frac{n}{2} - \left\{\frac{n}{4}\right\} - 1\right)!^2}$$

$$\times \sum_{i=0}^{\frac{n}{2} - \left\{\frac{n}{4}\right\} - 1} \binom{\frac{n}{2} - \left\{\frac{n}{4}\right\} - 1}{i} (-1)^i \, \frac{2\,B\left(\frac{n}{2} + \left(i + \left\{\frac{n}{4}\right\}\right) t + 1 \, , \, \frac{n}{2} - \left\{\frac{n}{4}\right\}\right)}{n - \left\{\frac{n}{4}\right\} - i + \left(i + \left\{\frac{n}{4}\right\}\right) t}$$

$$\times \left\{ \sum_{j=1}^{\frac{n}{2} - \left\{\frac{n}{4}\right\}} \frac{1}{n - \left\{\frac{n}{4}\right\} - j + 1 + \left(i + \left\{\frac{n}{4}\right\}\right) t} + \frac{1}{n - \left\{\frac{n}{4}\right\} - i + \left(i + \left\{\frac{n}{4}\right\}\right) t} \right\} ,$$

if $t \geq 1$.

For larger sample sizes we can use the normal distribution as an approximation to the distribution of T_n. In order to prove that T_n has a limiting normal distribution under the hypothesis of an exponential parent we consider the following lemma (cf. Chernoff *et al.*, 1967).

Lemma 2.1. *Let $Z_{i:n}$ be the i^{th} ascending order statistic of n i.i.d. standard exponential random variables $Z_1, ..., Z_n$. Then,*

$$P\left\{\tau_n^{-1}\sum_{i=1}^{n} a_{i:n}(Z_{i:n}-\mu_{i:n}) \leq t\right\} \xrightarrow[n\to+\infty]{} \Phi(t) ,$$

for every t, if and only if,

$$\max_{1\leq j\leq n} \tau_n^{-1}\,|b_{j,n}| \xrightarrow[n\to+\infty]{} 0 ,$$

where $\mu_{i:n} = E(Z_{i:n})$, $b_{j,n} = (n-j+1)^{-1}\sum_{i=j}^{n} a_{i:n}$ *and* $\tau_n^2 = \sum_{i=1}^{n} b_{i,n}^2$.

Using the lemma's notation for $X_{n-\{\frac{n}{4}\}+1:n} - X_{\frac{n+1}{2}:n}$ we have

$$a_{i:n} = \begin{cases} -1 , & i = \frac{n+1}{2} , \\ 1 , & i = n-\{\frac{n}{4}\}+1 , \\ 0 , & \text{elsewhere} , \end{cases}$$

and

$$b_{j,n} = \begin{cases} 0 , & 1 \leq j \leq \frac{n+1}{2} , \\ \dfrac{1}{n-j+1} , & \frac{n+1}{2}+1 \leq j \leq n-\{\frac{n}{4}\}+1 , \\ 0 , & n-\{\frac{n}{4}\}+2 \leq j \leq n . \end{cases}$$

Hence,

$$\tau_n^2 = \sum_{j=\frac{n+1}{2}+1}^{n-\{\frac{n}{4}\}+1} \frac{1}{(n-j+1)^2} = \sum_{k=\{\frac{n}{4}\}}^{\frac{n-1}{2}} \frac{1}{k^2} .$$

Applying the lemma we get

$$\tau_n^{-1}\Big(X_{n-\{\frac{n}{4}\}+1:n} - X_{\frac{n+1}{2}:n} - \lambda_n\Big) \xrightarrow[n\to+\infty]{d} Z \frown \text{Normal}(0,1) ,$$

where

$$\lambda_n = \mu_{n-\{\frac{n}{4}\}+1:n} - \mu_{\frac{n+1}{2}:n} = \sum_{k=\{\frac{n}{4}\}}^{\frac{n-1}{2}} \frac{1}{k} ,$$

and since $\frac{\ln(3/2)}{X_{\frac{n+1}{2}:n} - X_{\{\frac{n}{4}\}:n}}$ converges in probability to 1, it follows from Slutsky's theorem that

$$\ln(3/2)\;\tau_n^{-1}\Big(T_n - \frac{\lambda_n}{\ln(3/2)}\Big) \xrightarrow[n\to+\infty]{d} Z \frown \text{Normal}(0,1) .$$

However, simpler normalizing constants can be found. Since $\lambda_n \sim \ln 2$ and $\tau_n^2 \sim \frac{2}{n}$ we have

$$\ln(3/2)\sqrt{\frac{n}{2}}\left(T_n - \frac{\ln 2}{\ln(3/2)}\right) \xrightarrow[n\to\infty]{d} Z \frown \text{Normal}(0,1)\ .$$

On the other hand, it is quite straightforward to show that

$$\ln 2\ U_n - \ln n \xrightarrow[n\to\infty]{d} Y \frown \text{Gumbel}(0,1)$$

and

$$\ln 2\ V_n - \ln(n/2) \xrightarrow[n\to\infty]{d} Y \frown \text{Gumbel}(0,1)\ .$$

3. POWER, RESISTANCE AND ROBUSTNESS COMPARISON

The choice of the appropriate statistical test for a particular situation must be guided by a sensible criteriom. Usually, power considerations weight considerably in the decision process.

For inference purposes and comparison of the power functions we register on Table 1 the simulated critical points (based on 4999 simulations) of the sample distributions of T_n, U_n and V_n under the null hypothesis $\beta = 0$.

Table 1: Simulated critical points of T_n, U_n and V_n.

	$\boldsymbol{\alpha}$							
	.01	.025	.05	.1	.9	.95	.975	.99
T_{10}	.21*	.30*	.40*	.56*	4.90*	6.88*	9.37*	13.65*
U_{10}	1.53	1.70	1.87	2.12	7.62	9.53	11.68	14.97
V_{10}	.59	.79	1.00	1.29	8.00	10.38	12.99	18.07
T_{20}	.43*	.54*	.66*	.83*	3.89*	4.89*	5.97*	7.59*
U_{20}	2.13	2.37	2.65	2.98	8.50	10.08	11.82	13.95
V_{20}	1.18	1.47	1.75	2.11	8.23	10.02	11.95	14.72
T_{30}	.50*	.61*	.72*	.88*	3.30*	4.00*	4.74*	5.80*
U_{30}	2.53	2.91	3.17	3.54	9.11	10.47	12.05	13.87
V_{30}	1.62	1.96	2.25	2.66	8.57	10.10	11.64	13.57
T_{50}	.67	.79	.89	1.03	2.86	3.31	3.80	4.50
U_{50}	3.24	3.49	3.85	4.23	9.55	11.02	12.24	14.12
V_{50}	2.29	2.58	2.91	3.33	8.83	10.35	11.77	13.67
T_{100}	.87	.99	1.09	1.21	2.47	2.75	3.02	3.32
U_{100}	4.19	4.53	4.83	5.24	10.46	11.56	12.63	14.33
V_{100}	3.26	3.57	3.88	4.30	9.58	10.73	11.79	13.46
T_{250}	1.12	1.20	1.28	1.36	2.17	2.31	2.44	2.57
U_{250}	5.49	5.89	6.21	6.68	11.51	12.60	13.71	15.09
V_{250}	4.52	4.92	5.25	5.72	10.58	11.66	12.79	14.22

(* exact critical points)

Figures 1 and 2 show the simulated power functions (5 000 simulations) of T_n, U_n and V_n for a $\alpha = 0.05$ level right one-sided and two-sided tests and $n = 10, 20, 30, 50, 100, 250$.

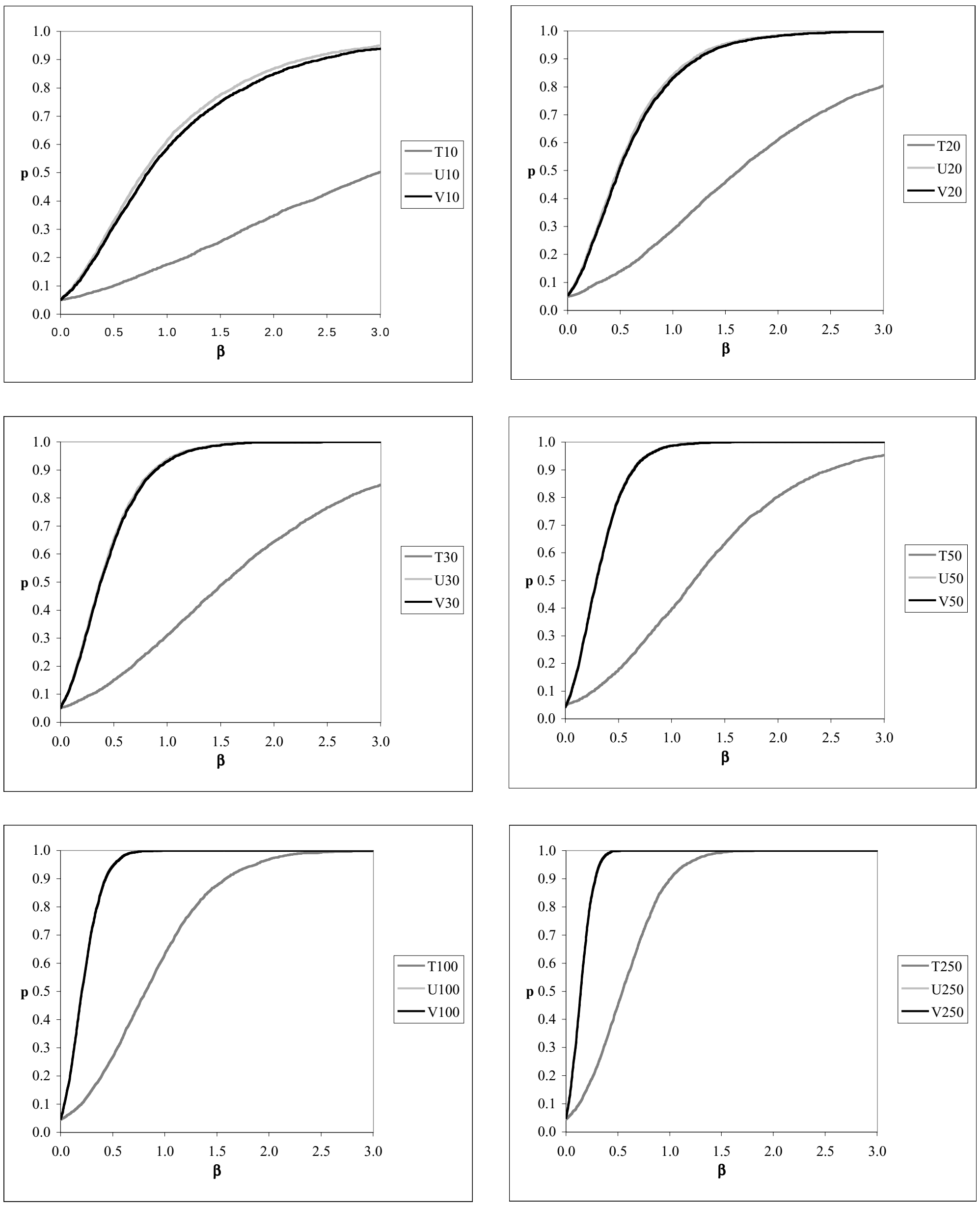

Figure 1: Power functions for a $\alpha = 0.05$ level right one-sided test.

From Figures 1 and 2 we observe that T_n performs quite badly in detecting departures from the exponential behaviour when compared with the other two tests. If we had to choose based exclusively on the power of the test, we would choose for smaller sample sizes ($n \leq 30$) U_n and for larger sample sizes U_n or V_n.

A comparison of the power of the three tests was also made for a $\alpha = 0.01$ level one-sided and two-sided tests, but the results are not presented here because they reveal a similar pattern as in the case $\alpha = 0.05$.

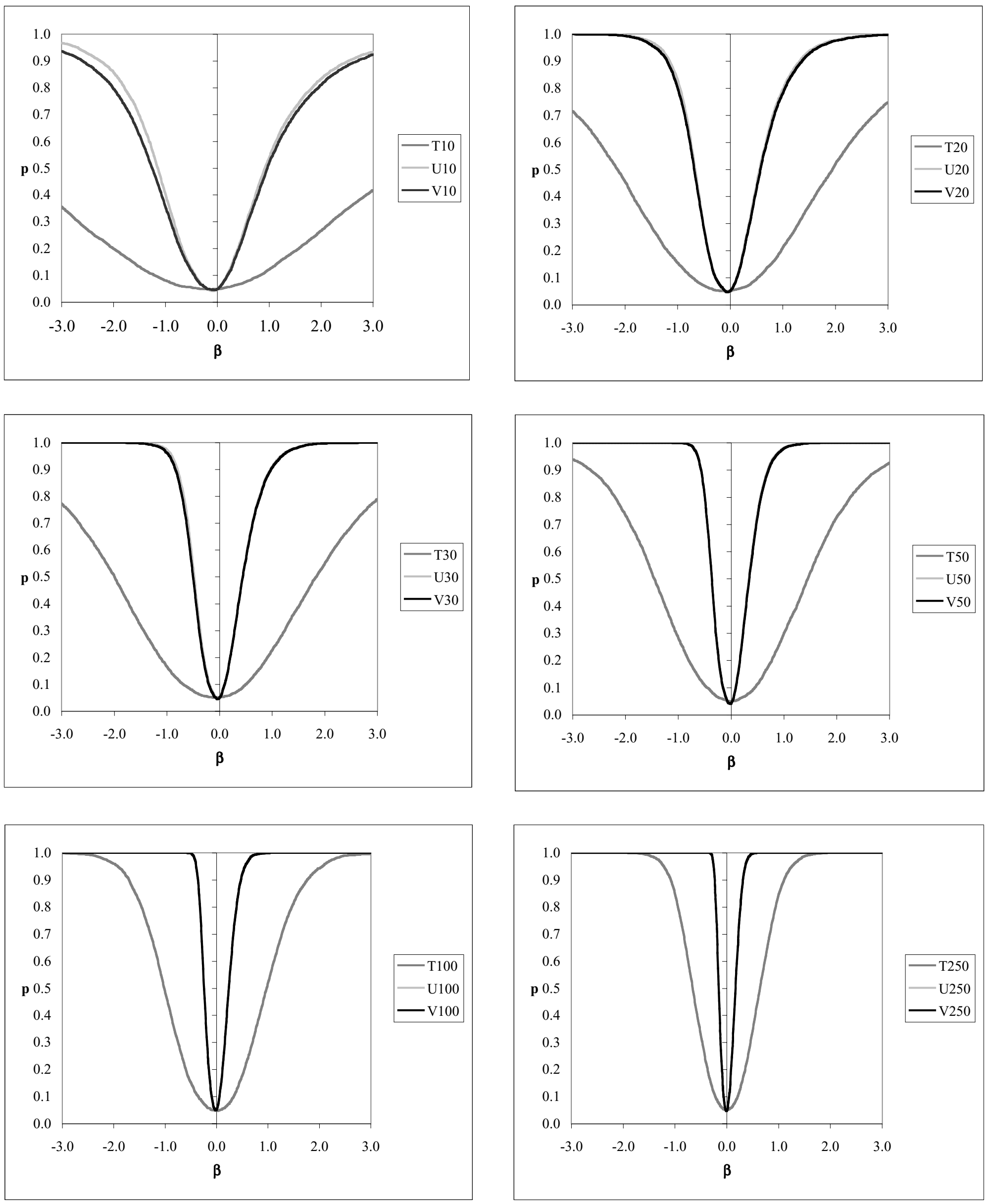

Figure 2: Power functions for a $\alpha = 0.05$ level two-sided test.

The power criteria can be pushed to a second place if we find stronger reasons which can sustain such a decision. In fact, the lesser power of T_n will be in a way compensated when we evaluate its performance after introducing disturbing observations in the sample (*e.g.*, an observation from an exponential population with a larger scale), or when we consider the sample from a mixture of exponentials, which is a variation (or contamination) of the "pure" exponential model. In order to compare the resistance and robustness of the tests we will evaluate the performance of each one in a broadened situation and mixture.

In an one broadened situation, which is the case presented here, we assume that the margins of the random vector $(X_1, ..., X_{n-1}, X^*)$ are independent, $X_1, ..., X_{n-1}$ are standard exponentials and X^* is an exponential variable with distribution function

$$F^*(x) = \left(1 - e^{-\frac{x+K-1}{K}}\right) I_{]1-K,+\infty[} \ .$$

In a mixture situation we assume that the random sample $(X_1, ..., X_n)$ is from a population with distribution function

$$F(x) \ = \ \left[(1-\theta)\left(1 - e^{-(x-\theta(1-K))}\right) + \theta\left(1 - e^{-\frac{x-\theta(1-K)}{K}}\right)\right] I_{]\theta(1-K),+\infty[} \ ,$$

where $0 < \theta < 1$ (θ is sometimes called the percentage contamination).

In robustness studies it is usual to consider $K = 3, 10$ and $\theta = 0.05, 0.1$ (cf. Hoaglin *et al.*, 1983). However, we will only show the results obtained for $K = 3, 10$ and $\theta = 0.05$, and for the classical level $\alpha = 0.05$.

In Tables 2 to 5 we indicate the probability of rejecting the exponential hypothesis, as well as the standard error of the estimates and the corresponding 95% confidence interval.

Table 2: Right one-sided test in an one broadened situation.

$K = 3$									
n	T_n	*s.e.*	95% C.I.	U_n	*s.e.*	95% C.I.	V_n	*s.e.*	95% C.I.
10	.050	.0031	[.044, .056]	.116	.0045	[.107, .125]	.070	.0036	[.063, .077]
20	.051	.0031	[.045, .057]	.099	.0042	[.091, .107]	.063	.0034	[.056, .070]
30	.048	.0030	[.042, .054]	.093	.0041	[.085, .101]	.067	.0035	[.060, .074]
50	.051	.0031	[.045, .057]	.086	.0040	[.078, .094]	.063	.0034	[.056, .070]
100	.053	.0032	[.047, .059]	.077	.0038	[.070, .084]	.053	.0032	[.047, .059]
250	.049	.0031	[.043, .055]	.079	.0038	[.072, .086]	.053	.0032	[.047, .059]

$K = 10$									
n	T_n	*s.e.*	95% C.I.	U_n	*s.e.*	95% C.I.	V_n	*s.e.*	95% C.I.
10	.055	.0032	[.049, .061]	.248	.0061	[.236, .260]	.186	.0055	[.175, .197]
20	.053	.0032	[.047, .059]	.239	.0060	[.227, .251]	.193	.0056	[.182, .204]
30	.050	.0031	[.044, .056]	.238	.0060	[.226, .250]	.203	.0057	[.192, .214]
50	.050	.0031	[.044, .056]	.226	.0059	[.214, .238]	.196	.0056	[.185, .207]
100	.054	.0032	[.048, .060]	.215	.0058	[.204, .226]	.185	.0055	[.174, .196]
250	.049	.0031	[.043, .055]	.206	.0057	[.195, .217]	.175	.0054	[.164, .186]

Table 3: Two-sided test in an one broadened situation.

K = 3									
n	T_n	*s.e.*	95% C.I.	U_n	*s.e.*	95% C.I.	V_n	*s.e.*	95% C.I.
10	.051	.0031	[.045, .057]	.085	.0039	[.077, .093]	.186	.0055	[.175, .197]
20	.052	.0031	[.046, .058]	.073	.0037	[.066, .080]	.273	.0063	[.261, .285]
30	.050	.0031	[.044, .056]	.078	.0038	[.071, .085]	.339	.0067	[.326, .352]
50	.054	.0032	[.048, .060]	.068	.0036	[.061, .075]	.381	.0069	[.368, .394]
100	.054	.0032	[.048, .060]	.070	.0036	[.063, .077]	.428	.0070	[.414, .442]
250	.056	.0033	[.050, .062]	.073	.0037	[.066, .080]	.467	.0071	[.453, .481]

K = 10									
n	T_n	*s.e.*	95% C.I.	U_n	*s.e.*	95% C.I.	V_n	*s.e.*	95% C.I.
10	.053	.0032	[.047, .059]	.205	.0057	[.194, .216]	.645	.0068	[.632, .658]
20	.052	.0031	[.046, .058]	.205	.0057	[.194, .216]	.701	.0065	[.688, .714]
30	.050	.0031	[.044, .056]	.208	.0057	[.197, .219]	.711	.0064	[.698, .724]
50	.054	.0032	[.048, .060]	.198	.0056	[.187, .209]	.723	.0063	[.711, .735]
100	.051	.0031	[.045, .057]	.203	.0057	[.192, .214]	.738	.0062	[.726, .750]
250	.056	.0033	[.050, .062]	.194	.0056	[.183, .205]	.750	.0061	[.738, .762]

Table 4: Right one-sided test in a 5% contamination situation.

K = 3									
n	T_n	*s.e.*	95% C.I.	U_n	*s.e.*	95% C.I.	V_n	*s.e.*	95% C.I.
10	.055	.0032	[.049, .061]	.142	.0049	[.132, .152]	.086	.0040	[.078, .094]
20	.056	.0033	[.050, .062]	.188	.0055	[.177, .199]	.119	.0046	[.110, .128]
30	.053	.0032	[.047, .059]	.217	.0058	[.206, .228]	.138	.0049	[.128, .148]
50	.058	.0033	[.052, .064]	.268	.0063	[.256, .280]	.182	.0055	[.171, .193]
100	.062	.0034	[.055, .069]	.405	.0069	[.391, .419]	.285	.0064	[.272, .298]
250	.064	.0035	[.057, .071]	.619	.0069	[.606, .632]	.471	.0071	[.457, .485]

K = 10									
n	T_n	*s.e.*	95% C.I.	U_n	*s.e.*	95% C.I.	V_n	*s.e.*	95% C.I.
10	.062	.0034	[.055, .069]	.459	.0070	[.445, .473]	.231	.0060	[.219, .243]
20	.069	.0036	[.062, .076]	.685	.0066	[.672, .698]	.382	.0069	[.369, .395]
30	.059	.0033	[.052, .066]	.816	.0055	[.805, .827]	.501	.0071	[.487, .515]
50	.065	.0035	[.058, .072]	.921	.0038	[.914, .928]	.683	.0066	[.670, .696]
100	.074	.0037	[.067, .081]	.992	.0013	[.990, .994]	.887	.0045	[.878, .896]
250	.092	.0041	[.084, .100]	1.000	.0000	—	.994	.0011	[.992, .996]

The analysis of the previous tables show that T_n is by far less sensitive to the disturbing observation, even when it comes from an exponential population with a standard deviation ten times greater than the standard deviation of the standard exponential. In other words this means that with T_n we will be rejecting a true null hypothesis with probability approximately equal to $\alpha = 0.05$. The same can be said when we consider that 5% of the observations are from an exponential population with standard deviation $K = 10$. Therefore the results confirm that T_n is more resistant and robust.

Table 5: Two-sided test in a 5% contamination situation.

$\boldsymbol{K=3}$									
$\boldsymbol{n}$	$\boldsymbol{T_n}$	***s.e.***	95% C.I.	$\boldsymbol{U_n}$	***s.e.***	95% C.I.	$\boldsymbol{V_n}$	***s.e.***	95% C.I.
10	.050	.0031	[.044, .056]	.106	.0044	[.097, .115]	.074	.0037	[.067, .081]
20	.053	.0032	[.047, .059]	.139	.0049	[.129, .149]	.093	.0041	[.085, .101]
30	.051	.0031	[.045, .057]	.160	.0052	[.150, .170]	.112	.0045	[.103, .121]
50	.052	.0031	[.046, .058]	.212	.0058	[.201, .223]	.143	.0050	[.133, .153]
100	.052	.0031	[.046, .058]	.333	.0067	[.320, .346]	.234	.0060	[.222, .246]
250	.050	.0031	[.044, .056]	.528	.0071	[.514, .542]	.385	.0069	[.372, .398]

$\boldsymbol{K=10}$									
$\boldsymbol{n}$	$\boldsymbol{T_n}$	***s.e.***	95% C.I.	$\boldsymbol{U_n}$	***s.e.***	95% C.I.	$\boldsymbol{V_n}$	***s.e.***	95% C.I.
10	.052	.0031	[.046, .058]	.545	.0070	[.531, .559]	.205	.0057	[.194, .216]
20	.053	.0032	[.047, .059]	.704	.0065	[.691, .717]	.345	.0067	[.332, .358]
30	.051	.0031	[.045, .057]	.805	.0056	[.794, .816]	.462	.0071	[.448, .476]
50	.054	.0032	[.048, .060]	.913	.0040	[.905, .921]	.645	.0068	[.632, .658]
100	.054	.0032	[.048, .060]	.989	.0015	[.986, .992]	.864	.0048	[.854, .874]
250	.060	.0034	[.053, .067]	1.000	.0000	—	.990	.0014	[.987, .993]

4. FINAL COMMENTS

It is important to use resistant and robust methods given the fact that: (i) classical techniques behave poorly when the general situation departs from the set of initial assumptions; (ii) in practice we never know the exact underlying conditions, specially when it is not so unlikely to admit the existence of disturbing data in the sample.

The conclusions of section 3 reinforce the general idea that resistant and robust methods are the best compromise possible for a large set of scenarios, although not necessarily the best ones for a very specific and limiting situation.

The analysis of the power function shows that the extreme order statistics carry important information for the issue at hand, and therefore trimming out 25% of the sample data may be too drastic. Unfortunately, there is no rule of thumb for an appropriate choice k in $T_{n(k)} = \frac{X_{n-k(n)+1:n}-M}{M-X_{k(n):n}}$ that optimizes results in what concerns power *and* resistance *and* robustness altogether.

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