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# DETECTING SOCIAL INTERACTIONS IN BIVARIATE PROBIT MODELS WITH AN ENDOGENOUS DUMMY VARIABLE: SOME SIMULATION RESULTS

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Abstract:

- This paper analyzes the possibility of detecting observable and non-observable social interactions in a bivariate probit model with an endogenous dummy regressor via Monte Carlo simulation. In small samples, we find severe size distortions of the Wald test and only a low probability of detecting observable and non-observable social interactions. In large samples, however, we find this test to be very powerful.

Key-Words:

- *parameter tests; bivariate probit model; Monte Carlo study.*

AMS Subject Classification:

- 62F03, 62P20.

## 1. INTRODUCTION

For the researcher, interactions between two persons, e.g., spouses or brothers and sisters may only partly be observable, due to measurement problems. Respondents may not wish to reveal the influence of the other person or may not be aware of them. In order to detect the neglected or non-observable interactions between the respective decision processes a bivariate probit model is recommendable (Jaenicke, 2004).

The identification of discrete choice models with social interactions is studied by Brock and Durlauf (2001, 2007). They show that it is possible to overcome Manski's (1993) famous reflection problem discussed in the context of reference group behaviour by using discrete choice models. In order to account for the neglected or non-observable interactions between discrete decisions in the respective decision processes, a bivariate probit model is a useful empirical description of this process. In this model, an endogenous dummy variable represents the binary decision of the peer that may have an influence on the second person. The bivariate probit model with an endogenous dummy variable is introduced by Maddala and Lee (1976) and belongs to a general class of simultaneous equation models discussed by Heckmann (1978), Maddala (1983), Wilde (2000, 2004), and Greene (2008).

In some applications of the bivariate probit model, e.g. Dean (1995), and Greene (1998) only small samples with 76 or 132 observations are available. However, even in data sets with 500, 1,000 or 2,000 observations, parameter tests may be crucial, as shown by Monfardini and Radici (2008).

Our intention is to find out for different sample sizes whether, in the presence of social interactions, it is possible to detect these interactions in a bivariate probit model with an endogenous dummy regressor. Hence we analyze the distribution of the estimated parameters and size and power of the usual $z$-coefficient test concerning the parameters of the observable and non-observable interactions, i.e. the endogenous dummy variable and the residual covariance between both equations of this bivariate probit model.

## 2. A BIVARIATE PROBIT MODEL OF SOCIAL INTERACTIONS

The maximum likelihood estimation of a bivariate probit model involves the numerical problem of the evaluation of double integrals over the normal distribution. This estimation procedure is implemented in several statistic software packages and widely used in practice. We use a two equation binary choice model with an endogenous dummy regressor, first proposed by Maddala and Lee (1976).

The regression equations of the individual $I$ and the peer $P$ are

$$\begin{aligned} Y_I^* &= X_I\beta_1 + Y_P\beta_2 + u_I\,, & Y_I &= 1 \quad \text{if } Y_I^* > 0, \quad 0 \text{ otherwise}\,, \\ Y_P^* &= X_P\gamma_1 + u_P\,, & Y_P &= 1 \quad \text{if } Y_P^* > 0, \quad 0 \text{ otherwise}\,, \\ [u_I, u_P] &\sim \Phi_2(0,0,1,1,\rho)\,, \end{aligned}$$

with the observable discrete choice behavior $Y$, latent variables $Y_I^*$, and exogenous variables $X$. The residual vector $[u_I, u_P]$ is bivariate normal distributed with $E(u_i) = 0$, $\text{var}(u_i) = 1$, $i = I, P$, and $\text{cov}(u_I, u_P) = \rho$. As a condition of identification, we only need exclusion restrictions if there is no variation of the exogenous regressors (Wilde, 2000).

In our model, the observable part of the social interactions, the influence of the decision of the peer $P$ on the behavior of the individual $I$ is tested by the hypothesis $H_0 : \beta_2 = 0$. The non-observable part of the social interactions may be revealed through the residual covariance structure. A residual covariance $\text{cov}(u_I, u_P)$, i.e. $\rho$, significantly different from zero, may serve as an indicator of unobserved social interactions between the two decisions or as an indicator of simultaneously neglected third-party effects. Restricting residual correlation of the bivariate probit model to zero may result in biased and inconsistent estimations (Murphy, 1995). Fitting separate probit models for the first- and the second decision equation can involve significant endogeneity biases in the estimation (Lollivier, 2001). The joint estimation of the two equations provides substantial efficiency gains compared to separate estimation based on two-stage technique.

## 3. MONTE CARLO RESULTS FOR THE BIVARIATE PROBIT MODEL

In a small Monte Carlo study, we analyze the size and the power of the usual $z$-coefficient tests concerning the parameters of the observable and non-observable social interactions, $\beta_2$ and $\rho$. The test statistics are $z(\beta_2) = \hat{\beta}_2 / se(\hat{\beta}_2)$ and $z(\rho) = \hat{\rho}/se(\hat{\rho})$ and their squares result in the standard Wald test (Greene 2008, p. 820).

The non-observable influences stem from missing variables. These may be uncorrelated, weakly or strongly correlated or identically for the two persons. To create the non-observable interactions, we use an omitted variable vector $[v_I, v_P] \sim \Phi_2(0,0,1,1,r)$ with $r \in [0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 1.0]$ in one set of experiments. In this case, the residuals $u_i$, $i = I, P$, are the sum $u_i = v_i + \varepsilon_i$ with $[\varepsilon_I, \varepsilon_P] \sim \Phi_2(0,0,1,1,0)$, therefore $[u_I, u_P] \sim \Phi_2(0,0,2,2,\frac{r}{2})$. Because the assumption of the unit residual variance $\text{var}(u_i)$ is not met, we expect some problems resulting from the misspecification of the model.

In the experiments with the extended model, we include $v_i$ as additional explanatory variables. In this case, the residuals are $u_i = \varepsilon_i$ and are independent normal distributed. Because of the independence of the residuals, $\rho = 0$, the model is overparametrized. Two single equation models would be more efficient. Anyway, since we do not know the true parameter set in the empirical research situation, in the simulation experiments we continue with bivariate probit models.

The variables $X_i$, $i = I, P$, are standard normal distributed, $X_i \sim N(0,1)$, $i = I, P$. All parameters $\beta$ and $\gamma$ in the omitted variable model and in the extended model are set equal to one. We use the econometric software package Limdep 7.0. It performs well in nonlinear estimation benchmark tests (McCullough, 1999). We estimate the bivariate models with the default settings of the procedure (algorithm: BFGS; maximum iterations: 100). The Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm is rather time consuming, but it shows a convergence rate of between 99.5 percent (in small data sets with 100 observations) and 100 percent (in data sets with 10,000 observations) in our Monte Carlo study. The number of replications in the Monte Carlo experiment is $N = 1{,}000$. The number of observations varies systematically from $T = 100$ to $T = 2{,}000$. In some experiments, we estimate data sets up to $T = 10{,}000$ and in one up to $T = 40{,}000$ observations. Due to the nonlinear estimation problem, experiments with these huge data sets are computational intensive, e.g., some experiments need around 84 hours on a Pentium(R), 3.2 GHz to estimate two bivariate probit models 1,000 times.

The estimated parameters $\widehat{\rho}$ show no severe bias. Figure 1 presents the density estimation with the Epanechnikov kernel function in the case that the true parameter $\rho = 0$. With increasing sample size from $T = 200$ to $T = 1{,}000$,

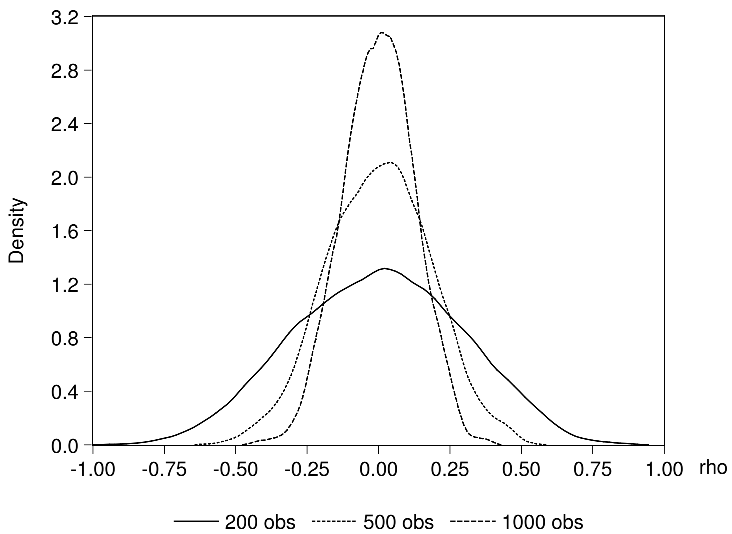


**Figure 1**: Kernel estimation, $\rho = 0$.

the dispersion of the estimated parameters becomes smaller. The parameters are distributed more or less symmetrically (with skewness $S_T$ between $-0.083$ and $-0.040$) and means (with $\overline{\widehat{\rho_T}}$ between $-0.001$ and $0.003$) very close to the theoretical value zero.

The picture changes if we assume with $\rho = 0.5$ strong residual correlation between both decision equations in small data sets. In the case of $T = 200$, we find with $\overline{\widehat{\rho_{200}}} = 0.485$ some deviations from the theoretical parameter value. In all three cases, the distribution of the estimated parameters $\widehat{\rho_{200}}$ is left skewed with a skewness $S_T$ between $-0.568$ and $-0.403$. In the $T = 200$ case, some $\widehat{\rho_{200}}$ are very close the theoretical limit of 1. In Figure 2, we present a Kernel density estimation for $\rho = 0.5$ and $T = 200$, $T = 500$ and $T = 1{,}000$.

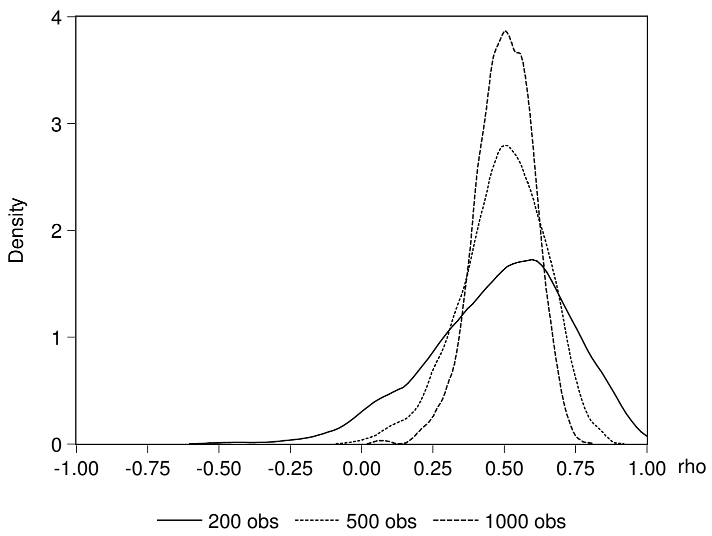


**Figure 2**: Kernel estimation, $\rho = 0.5$.

Looking at the $z$-statistics in the case $\rho = 0$, we find some indication that the $z$-statistics may not be normally distributed in small samples. To analyze this problem graphically, we compare the quantiles of these statistics with the standard normal distribution in Figure 3 for $T = 200, 500, 1000$ observations. Especially in the case of $\rho = 0$, $T = 200$ observations, we find some deviation from normality.

We start our analysis studying possible size distortions of the $z_\rho$-parameter test in the bivariate probit model with an endogenous dummy variable. The parameter describing the influence of the endogenous dummy variable is set equal to one, $\beta_2 = 1$. In Table 1 (see Appendix), we present percentiles of the simulated $z_\rho$-statistic, some descriptive statistics like mean, standard deviation, skewness, kurtosis, and the size of the test statistic with the nominal significance level equal to 1, 5 and 10 percent.

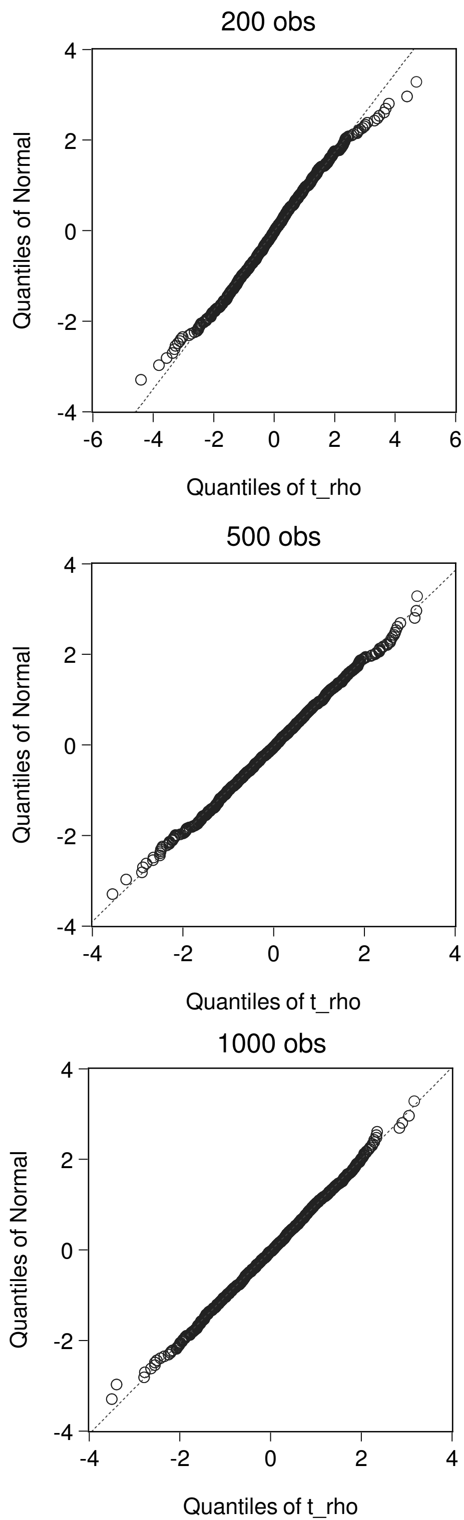


**Figure 3**: QQ-Plots for $t_\rho$, with $\rho = 0$ and $T = 200, 500, 1000$.

From Table 1, we see severe size distortions for the $z_\rho$-parameter test in small data sets. In the case of $T = 100, 150$ observations, the test statistic is excessively liberal. E.g., in the case of a nominal 5-percent level, the empirical size is more than twice as high. The deviations from the nominal level become stronger with a higher significance level. E.g., on the 1 percent level, the empirical size is 7 percent in the extreme case of $T = 100$ and 5 percent in the case of $T = 150$ observations. In these two cases the $z_\rho$-parameter test shows strong deviations from normality. The distribution is negatively skewed and reveals strong excess kurtosis. As expected, deviations from normality are not pronounced in the case of medium size or huge data sets.

In Table 2, we present a set of experiments using our bivariate probit model with an endogenous dummy variable but restricting the data generating mechanism to $\beta_2 = 0$, i.e., there is no endogenous dummy variable in the true model. The results are generally in line with the ones of Table 1, but with stronger deviations from normality in the small sample cases.

Using different correlation structures in Table 3 ($r = 0$) and Table 4 ($r = 1$), we analyse the small sample behaviour of the test concerning the observable interactions, the $z_{\beta_2}$-parameter test. We find again some size distortions for data sets with $T \leq 200$ observations but no systematic influence of the correlation $r$.

How will the size of the test statistic be affected if it is possible to include the omitted variable vector $[v_I, v_P]$ as additional regressors? Does the correlation $r$ have an influence on the outcome of the test? Comparing Tables 2 and 5, we see that the inclusion of additional explanatory variables makes the size distortions decrease only slightly. The comparison of Tables 5 and 6 makes clear that correlation between explanatory variables negatively affects the size of the $z_\rho$-test. In the case of $r = 1$, the empirical $p$-value of 10 percent is shifted to 23 percent in the $T = 100$ observation case. In this strong correlation case, we need more than $T = 2{,}000$ observations to obtain good size properties of the $z_\rho$-test. Summarizing Table 1 to 6, we find that in all small sample cases the test statistic is too liberal.

In the next simulation experiments, we have a look at the power properties of the test statistic. To answer the question whether it is possible to detect non-observable social interactions, we run 57 different Monte Carlo experiments. In the case of low residual correlation, the probability of finding significant interactions lies below one half in small or medium sized data sets. E.g., in the case of $r = 0.2$ and $T = 2{,}500$, only 35 percent of the $z_\rho$-tests are significant at the 10 percent level and in small data sets, e.g. $r = 0.2$ and $T = 100$, only 17 percent of the $z_\rho$-tests are significant at the 10 percent level. Taking into account the size distortions of the test statistic, the results will become even more unsatisfactionary. As expected, with more observations, and stronger correlation, the power increases. A power of at least 90 percent (at the nominal 10 percent level) can be found in the cases of $r = 0.2$ and $T = 15{,}000$ or $r = 1$ and $T = 2{,}500$.

The power properties of the $z_{\beta_2}$-test, presented in Table 8, are (at the 10 percent level) around one third in the small sample case and nearly 100 percent in $T \geq 1{,}000$ data sets. The changing of the correlation $r$ has more or less no influence on the Monte Carlo results.

The power can be dramatically increased if it is possible to include the neglected variable vector $[v_I, v_P]$. The results of these Monte Carlo experiments are presented in Table 9. The inclusion of these additional explanatory variables will shift the power properties significantly towards more than 50 percent even in the $T = 100$ case. With 250 observations, the power is close to the 90 percent level or higher. This is true although the extended model is overparametrized, because the true correlation-coefficient is zero in this model. In line with the results from Table 8, the results are nearly robust regarding the correlation structure $r$. Nevertheless, we have to take into account the too liberal behavior of our test in small samples.

## 4. CONCLUSIONS

In our paper, we find that the power of $z$-parameter tests concerning the residual correlation between the two decision equations in the bivariate probit model is very low in small samples. This is is especially true for weak correlations. The power of the parameter test concerning the endogenous dummy variable is around one third in small samples. If it is possible to find omitted variables, the power of this test can be increased notedly.

Additionally, our simulation results reveal severe over-rejection rates in small samples. Using a likelihood ratio test may result in better size behavior at least in medium sized samples (Monfardini and Radici, 2008).

From an empirical point of view, we may often fail to find significant social interactions in the data sets although they exist. An extensive search for omitted variables may therefore be essential to prove social interactions in empirical models.

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## REFERENCES


[1] Brock, W.A. and Durlauf, S.N. (2001). Discrete choice with social interactions, *Review of Economic Studies*, **68**, 235–260.

[2] Brock, W.A. and Durlauf, S.N. (2007). Identification of binary choice models with social interactions, *Journal of Econometrics*, **140**, 52–75.

[3] Dean, J.M. (1995). Market disruption and the incidence of VERs under the MFA, *Review of Economics and Statistics*, **77**, 383–388.

[4] Greene, W. (1998). Gender economics courses in liberal art colleges: Further results, *Journal of Economic Education*, **29**, 291–300.

[5] Greene, W.H. (2008). *Econometric Analysis*, 6th ed., Pearson, Prentice Hall.

[6] Heckman, J. (1978). Dummy endogenous variables in a simultaneous equation system, *Econometrica*, **9**, 255–268.

[7] Jaenicke, J. (2004). *Observable and Non-Observable Social Interactions in Labor Supply*, Discussion paper No. 2003/05, Rev. version, May 2004, University of Osnabrück.

[8] Lollivier, S. (2001). Endogénéité d'une variable explicative dichotomique dans le cadre d'un modèle probit bivarié, *Annales d'Économie et de Statistique*, **62**, 251–269.

[9] Maddala, G.S. (1983). *Limited Dependent and Quantitative Variables in Econometrics*, Cambridge University Press, Cambridge.

[10] Maddala, G.S. and Lee, L.-F. (1976). Recursive models with qualitative endogenous variables, *Annals of Economic and Social Measurement*, **5**, 525–545.

[11] Manski, C.F. (1993). Identification of endogenous social effects: the reflection problem, *Review of Economic Studies*, **60**, 531–542.

[12] McCullough, B.D. (1999). Econometric software reliability: EViews, LIMDEP, SHAZAM and TSP, *Journal of Applied Econometrics*, **14**, 191–202.

[13] Monfardini, C. and Radice, R. (2008). Testing exogeneity in the bivariate probit model: A Monte Carlo study, *Oxford Bulletin of Economics and Statistics*, **70**, 271–282.

[14] Murphy, A. (1995). Female labour force participation and unemployment in Northern Ireland: Religion and family effects, *Economic and Social Review*, **27**, 67–84.

[15] Wilde, J. (2000). Identification of multiple equation probit models with endogenous dummy regressors, *Economics Letters*, **69**, 309–312.

[16] Wilde, J. (2004). Estimating multiple equation hybrid models with endogenous dummy regressors, *Statistica Neerlandica*, **58**, 296–312.

# Appendix

**Table 1:** **Finite sample behavior of the z_ρ - parameter test concerning the non-observable interactions in the case of r=0 and β2=1**

| | | percentiles | | | | | | | | | | | | | | | | | p-value | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **r** | **T** | **0.01** | **0.05** | **0.1** | **0.2** | **0.3** | **0.4** | **0.5** | **0.6** | **0.7** | **0.8** | **0.9** | **0.95** | **0.99** | **mean** | **std.** | **skew.** | **kurt.** | **0.01** | **0.05** | **0.1** |
| 0.0 | 100 | -4.07 | -2.09 | -1.50 | -0.91 | -0.53 | -0.27 | -0.02 | 0.21 | 0.50 | 0.80 | 1.51 | 2.11 | 4.85 | -0.04 | 1.60 | -1.47 | 23.14 | 0.07 | 0.12 | 0.17 |
| 0.0 | 150 | -4.00 | -1.97 | -1.46 | -0.82 | -0.51 | -0.26 | 0.00 | 0.21 | 0.47 | 0.81 | 1.35 | 1.87 | 3.85 | -0.06 | 1.42 | -2.04 | 22.25 | 0.05 | 0.10 | 0.15 |
| 0.0 | 200 | -3.04 | -1.78 | -1.36 | -0.87 | -0.53 | -0.24 | 0.01 | 0.27 | 0.53 | 0.91 | 1.41 | 1.90 | 3.01 | 0.02 | 1.14 | 0.09 | 3.94 | 0.03 | 0.08 | 0.14 |
| 0.0 | 250 | -2.36 | -1.60 | -1.24 | -0.81 | -0.49 | -0.25 | 0.00 | 0.24 | 0.55 | 0.86 | 1.44 | 1.99 | 2.91 | 0.05 | 1.06 | 0.36 | 3.57 | 0.02 | 0.07 | 0.12 |
| 0.0 | 500 | -2.50 | -1.61 | -1.24 | -0.82 | -0.50 | -0.25 | 0.03 | 0.29 | 0.56 | 0.86 | 1.34 | 1.73 | 2.59 | 0.03 | 1.03 | 0.00 | 3.18 | 0.02 | 0.06 | 0.10 |
| 0.0 | 750 | -2.54 | -1.81 | -1.29 | -0.84 | -0.56 | -0.23 | 0.02 | 0.27 | 0.54 | 0.90 | 1.33 | 1.71 | 2.54 | 0.01 | 1.04 | -0.01 | 3.07 | 0.02 | 0.06 | 0.12 |
| 0.0 | 1000 | -2.35 | -1.58 | -1.27 | -0.82 | -0.50 | -0.23 | 0.04 | 0.29 | 0.54 | 0.82 | 1.29 | 1.67 | 2.24 | 0.02 | 0.99 | -0.04 | 3.07 | 0.01 | 0.05 | 0.09 |
| 0.0 | 1500 | -2.58 | -1.66 | -1.24 | -0.83 | -0.52 | -0.20 | 0.04 | 0.29 | 0.56 | 0.87 | 1.34 | 1.63 | 2.38 | 0.03 | 1.02 | -0.11 | 3.25 | 0.02 | 0.04 | 0.10 |
| 0.0 | 2000 | -2.22 | -1.63 | -1.21 | -0.79 | -0.51 | -0.22 | 0.07 | 0.32 | 0.58 | 0.87 | 1.32 | 1.63 | 2.25 | 0.04 | 0.98 | -0.08 | 2.89 | 0.01 | 0.04 | 0.09 |
| 0.0 | 2500 | -2.21 | -1.59 | -1.23 | -0.79 | -0.51 | -0.25 | 0.03 | 0.28 | 0.54 | 0.83 | 1.30 | 1.61 | 2.42 | 0.03 | 0.98 | 0.09 | 3.04 | 0.01 | 0.05 | 0.09 |
| 0.0 | 5000 | -2.20 | -1.61 | -1.17 | -0.76 | -0.44 | -0.22 | 0.02 | 0.27 | 0.53 | 0.90 | 1.26 | 1.62 | 2.35 | 0.04 | 0.97 | 0.02 | 2.97 | 0.01 | 0.04 | 0.09 |
| 0.0 | 7500 | -2.15 | -1.57 | -1.25 | -0.78 | -0.45 | -0.20 | 0.07 | 0.31 | 0.55 | 0.85 | 1.24 | 1.50 | 2.44 | 0.04 | 0.96 | -0.03 | 2.91 | 0.01 | 0.04 | 0.09 |
| 0.0 | 10000 | -2.33 | -1.60 | -1.20 | -0.80 | -0.44 | -0.20 | 0.07 | 0.29 | 0.55 | 0.86 | 1.28 | 1.63 | 2.30 | 0.04 | 0.98 | -0.01 | 3.09 | 0.01 | 0.05 | 0.09 |

**Table 2:** **Finite sample behavior of the z_ρ - parameter test concerning the non-observable interactions in the case of r=0 and β2=0**

| | | percentiles | | | | | | | | | | | | | | | | | p-value | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **r** | **T** | **0.01** | **0.05** | **0.1** | **0.2** | **0.3** | **0.4** | **0.5** | **0.6** | **0.7** | **0.8** | **0.9** | **0.95** | **0.99** | **mean** | **std.** | **skew.** | **kurt.** | **0.01** | **0.05** | **0.1** |
| 0.0 | 100 | -4.22 | -1.97 | -1.48 | -0.90 | -0.57 | -0.30 | -0.05 | 0.18 | 0.48 | 0.83 | 1.38 | 2.02 | 4.09 | -0.08 | 1.56 | -2.82 | 31.83 | 0.06 | 0.10 | 0.16 |
| 0.0 | 150 | -3.06 | -2.00 | -1.47 | -0.88 | -0.54 | -0.25 | 0.01 | 0.23 | 0.49 | 0.87 | 1.46 | 1.94 | 2.63 | -0.07 | 1.90 | -15.57 | 387.95 | 0.03 | 0.10 | 0.15 |
| 0.0 | 200 | -2.81 | -1.76 | -1.37 | -0.85 | -0.52 | -0.25 | 0.02 | 0.26 | 0.50 | 0.83 | 1.38 | 1.90 | 2.97 | 0.01 | 1.11 | 0.05 | 3.73 | 0.03 | 0.08 | 0.14 |
| 0.0 | 250 | -2.52 | -1.55 | -1.21 | -0.78 | -0.48 | -0.25 | 0.02 | 0.29 | 0.59 | 0.90 | 1.34 | 1.76 | 3.14 | 0.06 | 1.05 | 0.22 | 4.31 | 0.02 | 0.06 | 0.10 |
| 0.0 | 500 | -2.59 | -1.65 | -1.24 | -0.78 | -0.48 | -0.24 | 0.02 | 0.27 | 0.56 | 0.92 | 1.31 | 1.67 | 2.50 | 0.03 | 1.02 | -0.13 | 3.39 | 0.02 | 0.05 | 0.10 |
| 0.0 | 750 | -2.50 | -1.63 | -1.25 | -0.85 | -0.55 | -0.27 | 0.01 | 0.24 | 0.54 | 0.86 | 1.35 | 1.77 | 2.51 | 0.01 | 1.05 | 0.10 | 3.36 | 0.02 | 0.07 | 0.11 |
| 0.0 | 1000 | -2.41 | -1.54 | -1.22 | -0.82 | -0.52 | -0.24 | 0.02 | 0.28 | 0.55 | 0.86 | 1.35 | 1.65 | 2.33 | 0.03 | 0.99 | -0.01 | 3.02 | 0.01 | 0.04 | 0.09 |
| 0.0 | 1500 | -2.42 | -1.60 | -1.28 | -0.86 | -0.54 | -0.26 | 0.04 | 0.29 | 0.59 | 0.89 | 1.39 | 1.70 | 2.29 | 0.03 | 1.03 | -0.03 | 3.10 | 0.01 | 0.04 | 0.10 |
| 0.0 | 2000 | -2.48 | -1.69 | -1.28 | -0.84 | -0.49 | -0.18 | 0.09 | 0.36 | 0.59 | 0.94 | 1.35 | 1.68 | 2.45 | 0.05 | 1.03 | -0.10 | 3.05 | 0.01 | 0.05 | 0.11 |
| 0.0 | 2500 | -2.20 | -1.62 | -1.25 | -0.80 | -0.48 | -0.26 | 0.01 | 0.29 | 0.56 | 0.90 | 1.32 | 1.69 | 2.43 | 0.03 | 1.00 | 0.07 | 2.94 | 0.01 | 0.05 | 0.10 |
| 0.0 | 5000 | -2.32 | -1.63 | -1.25 | -0.80 | -0.47 | -0.20 | 0.06 | 0.31 | 0.58 | 0.91 | 1.27 | 1.57 | 2.28 | 0.04 | 0.98 | -0.09 | 2.90 | 0.01 | 0.04 | 0.09 |
| 0.0 | 7500 | -2.25 | -1.56 | -1.20 | -0.79 | -0.49 | -0.24 | 0.06 | 0.33 | 0.55 | 0.81 | 1.30 | 1.61 | 2.28 | 0.04 | 0.97 | -0.01 | 2.95 | 0.01 | 0.04 | 0.08 |
| 0.0 | 10000 | -2.31 | -1.51 | -1.23 | -0.82 | -0.53 | -0.24 | 0.07 | 0.32 | 0.56 | 0.87 | 1.31 | 1.69 | 2.34 | 0.03 | 0.99 | -0.06 | 3.05 | 0.01 | 0.05 | 0.09 |

**Table 3:** **Finite sample behavior of the z_β2 - parameter test concerning the observable interactions in the case of r=0 and β2=0**

| | | percentiles | | | | | | | | | | | | | | | | | p-value | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **r** | **T** | **0.01** | **0.05** | **0.1** | **0.2** | **0.3** | **0.4** | **0.5** | **0.6** | **0.7** | **0.8** | **0.9** | **0.95** | **0.99** | **mean** | **std.** | **skew.** | **kurt.** | **0.01** | **0.05** | **0.1** |
| 0.0 | 100 | -4.76 | -1.95 | -1.37 | -0.84 | -0.52 | -0.28 | -0.02 | 0.26 | 0.54 | 0.88 | 1.38 | 1.86 | 3.15 | -0.05 | 1.33 | -1.32 | 11.06 | 0.05 | 0.09 | 0.14 |
| 0.0 | 150 | -2.71 | -1.76 | -1.34 | -0.87 | -0.56 | -0.27 | -0.01 | 0.25 | 0.55 | 0.91 | 1.42 | 1.89 | 2.65 | 0.00 | 1.19 | -0.83 | 12.08 | 0.03 | 0.08 | 0.13 |
| 0.0 | 200 | -2.75 | -1.81 | -1.46 | -0.92 | -0.53 | -0.25 | 0.01 | 0.27 | 0.59 | 0.88 | 1.35 | 1.73 | 2.57 | -0.02 | 1.09 | -0.11 | 3.20 | 0.02 | 0.07 | 0.13 |
| 0.0 | 250 | -2.49 | -1.77 | -1.38 | -0.89 | -0.56 | -0.26 | -0.05 | 0.23 | 0.50 | 0.77 | 1.17 | 1.54 | 2.42 | -0.06 | 1.01 | -0.13 | 3.25 | 0.01 | 0.06 | 0.10 |
| 0.0 | 500 | -2.33 | -1.69 | -1.34 | -0.93 | -0.61 | -0.29 | 0.00 | 0.22 | 0.46 | 0.78 | 1.24 | 1.60 | 2.47 | -0.04 | 1.02 | 0.14 | 3.24 | 0.01 | 0.06 | 0.10 |
| 0.0 | 750 | -2.76 | -1.61 | -1.37 | -0.90 | -0.59 | -0.27 | 0.07 | 0.30 | 0.57 | 0.87 | 1.35 | 1.79 | 2.54 | 0.00 | 1.06 | -0.02 | 3.07 | 0.02 | 0.06 | 0.10 |
| 0.0 | 1000 | -2.28 | -1.63 | -1.31 | -0.90 | -0.59 | -0.31 | 0.03 | 0.27 | 0.53 | 0.81 | 1.22 | 1.61 | 2.37 | -0.02 | 1.00 | 0.06 | 2.97 | 0.01 | 0.05 | 0.09 |
| 0.0 | 1500 | -2.31 | -1.73 | -1.37 | -0.90 | -0.59 | -0.25 | -0.03 | 0.25 | 0.54 | 0.86 | 1.28 | 1.62 | 2.22 | -0.03 | 1.03 | 0.04 | 3.00 | 0.01 | 0.06 | 0.11 |
| 0.0 | 2000 | -2.27 | -1.66 | -1.33 | -0.89 | -0.60 | -0.29 | -0.03 | 0.21 | 0.48 | 0.80 | 1.28 | 1.68 | 2.24 | -0.03 | 1.01 | 0.11 | 3.23 | 0.01 | 0.04 | 0.11 |
| 0.0 | 2500 | -2.33 | -1.69 | -1.32 | -0.82 | -0.49 | -0.25 | 0.00 | 0.22 | 0.49 | 0.84 | 1.25 | 1.56 | 2.30 | -0.01 | 0.99 | -0.06 | 3.12 | 0.01 | 0.05 | 0.09 |
| 0.0 | 5000 | -2.28 | -1.59 | -1.26 | -0.83 | -0.58 | -0.30 | -0.06 | 0.23 | 0.52 | 0.83 | 1.29 | 1.66 | 2.42 | -0.01 | 0.99 | 0.12 | 2.86 | 0.01 | 0.05 | 0.10 |
| 0.0 | 7500 | -2.22 | -1.55 | -1.21 | -0.82 | -0.53 | -0.30 | -0.03 | 0.20 | 0.47 | 0.77 | 1.24 | 1.62 | 2.19 | -0.02 | 0.96 | 0.05 | 3.03 | 0.01 | 0.05 | 0.09 |
| 0.0 | 10000 | -2.26 | -1.63 | -1.20 | -0.83 | -0.54 | -0.27 | -0.04 | 0.21 | 0.50 | 0.77 | 1.21 | 1.60 | 2.55 | -0.02 | 0.98 | 0.14 | 3.29 | 0.01 | 0.05 | 0.10 |

**Table 4:** **Finite sample behavior of the z_β2 - parameter test concerning the observable interactions in the case of r=1 and β2=0**

| | | percentiles | | | | | | | | | | | | | | | | | p-value | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **r** | **T** | **0.01** | **0.05** | **0.1** | **0.2** | **0.3** | **0.4** | **0.5** | **0.6** | **0.7** | **0.8** | **0.9** | **0.95** | **0.99** | **mean** | **std.** | **skew.** | **kurt.** | **0.01** | **0.05** | **0.10** |
| 1.0 | 100 | -4.96 | -2.50 | -1.64 | -0.93 | -0.59 | -0.29 | -0.03 | 0.21 | 0.50 | 0.79 | 1.18 | 1.58 | 2.13 | -0.17 | 1.28 | -1.23 | 6.76 | 0.11 | 0.09 | 0.14 |
| 1.0 | 150 | -4.32 | -2.22 | -1.60 | -0.96 | -0.59 | -0.29 | -0.01 | 0.23 | 0.51 | 0.82 | 1.23 | 1.62 | 2.21 | -0.14 | 1.25 | -1.34 | 7.78 | 0.08 | 0.09 | 0.14 |
| 1.0 | 200 | -4.12 | -2.04 | -1.58 | -1.01 | -0.60 | -0.25 | 0.03 | 0.25 | 0.53 | 0.81 | 1.21 | 1.48 | 2.21 | -0.13 | 1.23 | -1.61 | 10.72 | 0.03 | 0.08 | 0.13 |
| 1.0 | 250 | -3.15 | -1.92 | -1.38 | -0.93 | -0.60 | -0.27 | -0.04 | 0.19 | 0.47 | 0.77 | 1.10 | 1.41 | 2.08 | -0.12 | 1.05 | -0.51 | 3.89 | 0.03 | 0.06 | 0.10 |
| 1.0 | 500 | -2.71 | -1.79 | -1.37 | -0.94 | -0.60 | -0.28 | -0.02 | 0.19 | 0.43 | 0.72 | 1.18 | 1.52 | 2.36 | -0.09 | 1.01 | -0.14 | 3.25 | 0.02 | 0.06 | 0.10 |
| 1.0 | 750 | -2.90 | -1.72 | -1.33 | -0.90 | -0.54 | -0.27 | -0.01 | 0.25 | 0.51 | 0.79 | 1.28 | 1.65 | 2.31 | -0.04 | 1.05 | -0.30 | 3.55 | 0.02 | 0.06 | 0.11 |
| 1.0 | 1000 | -2.52 | -1.72 | -1.33 | -0.94 | -0.60 | -0.30 | 0.01 | 0.25 | 0.48 | 0.77 | 1.17 | 1.52 | 2.12 | -0.06 | 1.00 | -0.12 | 3.01 | 0.01 | 0.06 | 0.09 |
| 1.0 | 1500 | -2.53 | -1.77 | -1.39 | -0.93 | -0.62 | -0.30 | -0.04 | 0.26 | 0.54 | 0.81 | 1.21 | 1.53 | 2.25 | -0.05 | 1.02 | -0.07 | 3.05 | 0.01 | 0.05 | 0.10 |
| 1.0 | 2000 | -2.42 | -1.70 | -1.32 | -0.92 | -0.62 | -0.31 | -0.06 | 0.22 | 0.48 | 0.81 | 1.27 | 1.65 | 2.18 | -0.05 | 1.01 | -0.01 | 3.10 | 0.01 | 0.05 | 0.11 |
| 1.0 | 2500 | -2.49 | -1.70 | -1.31 | -0.85 | -0.51 | -0.24 | -0.02 | 0.26 | 0.50 | 0.85 | 1.23 | 1.61 | 2.14 | -0.03 | 0.99 | -0.14 | 3.12 | 0.01 | 0.05 | 0.10 |
| 1.0 | 5000 | -2.22 | -1.65 | -1.20 | -0.84 | -0.54 | -0.29 | -0.03 | 0.23 | 0.50 | 0.81 | 1.28 | 1.65 | 2.43 | -0.01 | 0.98 | 0.06 | 2.89 | 0.01 | 0.05 | 0.10 |
| 1.0 | 7500 | -2.36 | -1.60 | -1.22 | -0.79 | -0.53 | -0.27 | -0.02 | 0.22 | 0.49 | 0.79 | 1.26 | 1.58 | 2.18 | -0.02 | 0.96 | -0.02 | 2.99 | 0.01 | 0.05 | 0.09 |
| 1.0 | 10000 | -2.25 | -1.66 | -1.24 | -0.77 | -0.55 | -0.26 | -0.03 | 0.22 | 0.48 | 0.79 | 1.22 | 1.61 | 2.46 | -0.01 | 0.98 | 0.08 | 3.14 | 0.01 | 0.04 | 0.10 |

**Table 5:** **Finite sample behavior of the z_ρ - parameter test in the case of r=0, β2=0 and additional explanatory variables**

| | | percentiles | | | | | | | | | | | | | | | | | p-value | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **r** | **T** | **0.01** | **0.05** | **0.1** | **0.2** | **0.3** | **0.4** | **0.5** | **0.6** | **0.7** | **0.8** | **0.9** | **0.95** | **0.99** | **mean** | **std.** | **skew.** | **kurt.** | **0.01** | **0.05** | **0.1** |
| 0.0 | 100 | -4.14 | -2.27 | -1.67 | -0.95 | -0.58 | -0.31 | -0.06 | 0.21 | 0.50 | 0.86 | 1.42 | 1.99 | 3.65 | -0.08 | 1.39 | -0.66 | 7.78 | 0.06 | 0.12 | 0.17 |
| 0.0 | 150 | -3.34 | -2.00 | -1.46 | -0.90 | -0.55 | -0.27 | -0.04 | 0.22 | 0.49 | 0.83 | 1.31 | 1.73 | 2.93 | -0.06 | 1.43 | -1.37 | 40.10 | 0.04 | 0.09 | 0.13 |
| 0.0 | 200 | -3.16 | -1.94 | -1.35 | -0.84 | -0.52 | -0.23 | 0.01 | 0.24 | 0.50 | 0.88 | 1.38 | 1.76 | 2.57 | -0.02 | 1.14 | -0.20 | 4.29 | 0.03 | 0.08 | 0.13 |
| 0.0 | 250 | -2.91 | -1.73 | -1.30 | -0.82 | -0.51 | -0.23 | 0.01 | 0.27 | 0.54 | 0.85 | 1.36 | 1.74 | 2.49 | 0.00 | 1.11 | -0.53 | 7.04 | 0.02 | 0.07 | 0.12 |
| 0.0 | 500 | -2.23 | -1.68 | -1.22 | -0.80 | -0.48 | -0.23 | 0.00 | 0.27 | 0.54 | 0.88 | 1.33 | 1.73 | 2.35 | 0.03 | 1.01 | 0.01 | 3.03 | 0.01 | 0.05 | 0.11 |
| 0.0 | 750 | -2.33 | -1.66 | -1.24 | -0.81 | -0.52 | -0.24 | 0.02 | 0.25 | 0.47 | 0.77 | 1.23 | 1.60 | 2.29 | -0.01 | 0.98 | -0.08 | 3.43 | 0.01 | 0.05 | 0.10 |
| 0.0 | 1000 | -2.43 | -1.69 | -1.29 | -0.84 | -0.47 | -0.26 | 0.02 | 0.29 | 0.53 | 0.86 | 1.32 | 1.68 | 2.46 | 0.02 | 1.02 | -0.04 | 3.01 | 0.01 | 0.06 | 0.11 |
| 0.0 | 1500 | -2.27 | -1.76 | -1.34 | -0.82 | -0.50 | -0.25 | 0.04 | 0.30 | 0.54 | 0.86 | 1.31 | 1.67 | 2.45 | 0.01 | 1.02 | 0.04 | 2.94 | 0.01 | 0.06 | 0.11 |
| 0.0 | 2000 | -2.35 | -1.62 | -1.27 | -0.81 | -0.54 | -0.25 | 0.03 | 0.27 | 0.54 | 0.86 | 1.33 | 1.73 | 2.65 | 0.03 | 1.02 | 0.11 | 3.10 | 0.02 | 0.05 | 0.11 |
| 0.0 | 2500 | -2.30 | -1.61 | -1.24 | -0.85 | -0.52 | -0.28 | 0.00 | 0.29 | 0.55 | 0.88 | 1.35 | 1.69 | 2.30 | 0.02 | 1.01 | 0.12 | 2.97 | 0.01 | 0.04 | 0.11 |
| 0.0 | 5000 | -2.57 | -1.57 | -1.30 | -0.82 | -0.48 | -0.21 | 0.03 | 0.30 | 0.55 | 0.84 | 1.26 | 1.78 | 2.50 | 0.03 | 1.01 | 0.01 | 3.13 | 0.02 | 0.06 | 0.10 |
| 0.0 | 7500 | -2.21 | -1.55 | -1.25 | -0.79 | -0.52 | -0.23 | 0.04 | 0.27 | 0.53 | 0.85 | 1.36 | 1.71 | 2.40 | 0.03 | 0.99 | 0.06 | 3.01 | 0.01 | 0.05 | 0.11 |
| 0.0 | 10000 | -2.38 | -1.63 | -1.26 | -0.81 | -0.48 | -0.19 | 0.03 | 0.30 | 0.56 | 0.91 | 1.36 | 1.71 | 2.55 | 0.05 | 1.03 | -0.02 | 3.10 | 0.01 | 0.06 | 0.11 |

**Table 6:** **Finite sample behavior of the z_ρ - parameter test in the case of r=1, β2=0 and additional explanatory variables**

| | | percentiles | | | | | | | | | | | | | | | | | p-value | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **r** | **T** | **0.01** | **0.05** | **0.1** | **0.2** | **0.3** | **0.4** | **0.5** | **0.6** | **0.7** | **0.8** | **0.9** | **0.95** | **0.99** | **mean** | **std.** | **skew.** | **kurt.** | **0.01** | **0.05** | **0.1** |
| 1.0 | 100 | -8.40 | -3.45 | -1.81 | -1.01 | -0.60 | -0.29 | -0.03 | 0.24 | 0.49 | 0.93 | 1.73 | 2.46 | 4.83 | -0.15 | 1.98 | -1.56 | 12.84 | 0.11 | 0.17 | 0.23 |
| 1.0 | 150 | -5.33 | -2.35 | -1.59 | -0.93 | -0.57 | -0.24 | 0.01 | 0.27 | 0.54 | 0.93 | 1.58 | 2.15 | 4.22 | -0.05 | 1.62 | -1.42 | 14.00 | 0.08 | 0.13 | 0.19 |
| 1.0 | 200 | -6.64 | -2.28 | -1.59 | -0.98 | -0.56 | -0.28 | -0.01 | 0.22 | 0.50 | 0.92 | 1.52 | 2.23 | 3.43 | -0.10 | 1.71 | -2.92 | 28.76 | 0.07 | 0.13 | 0.18 |
| 1.0 | 250 | -4.44 | -1.96 | -1.39 | -0.84 | -0.54 | -0.26 | -0.02 | 0.28 | 0.57 | 0.92 | 1.52 | 2.01 | 2.84 | -0.05 | 1.60 | -4.66 | 50.76 | 0.04 | 0.10 | 0.15 |
| 1.0 | 500 | -2.98 | -1.78 | -1.39 | -0.84 | -0.53 | -0.22 | 0.01 | 0.27 | 0.52 | 0.84 | 1.33 | 1.77 | 2.66 | 0.00 | 1.09 | -0.11 | 3.75 | 0.03 | 0.08 | 0.12 |
| 1.0 | 750 | -2.65 | -1.89 | -1.41 | -0.89 | -0.50 | -0.25 | 0.01 | 0.25 | 0.50 | 0.88 | 1.27 | 1.77 | 2.74 | -0.02 | 1.10 | -0.09 | 3.63 | 0.02 | 0.07 | 0.13 |
| 1.0 | 1000 | -2.62 | -1.73 | -1.38 | -0.87 | -0.51 | -0.22 | 0.04 | 0.31 | 0.58 | 0.82 | 1.30 | 1.73 | 2.41 | 0.01 | 1.05 | -0.16 | 3.46 | 0.02 | 0.07 | 0.11 |
| 1.0 | 1500 | -2.88 | -1.70 | -1.31 | -0.85 | -0.47 | -0.22 | 0.01 | 0.29 | 0.55 | 0.85 | 1.30 | 1.72 | 2.62 | 0.01 | 1.07 | -0.14 | 3.77 | 0.02 | 0.07 | 0.12 |
| 1.0 | 2000 | -2.54 | -1.72 | -1.34 | -0.77 | -0.46 | -0.21 | 0.03 | 0.29 | 0.55 | 0.89 | 1.34 | 1.80 | 2.69 | 0.03 | 1.07 | 0.03 | 3.72 | 0.02 | 0.07 | 0.12 |
| 1.0 | 2500 | -2.49 | -1.55 | -1.24 | -0.81 | -0.47 | -0.24 | 0.01 | 0.24 | 0.52 | 0.84 | 1.30 | 1.74 | 2.58 | 0.03 | 1.02 | 0.15 | 3.54 | 0.02 | 0.06 | 0.10 |
| 1.0 | 5000 | -2.34 | -1.51 | -1.18 | -0.82 | -0.50 | -0.24 | -0.01 | 0.27 | 0.53 | 0.90 | 1.38 | 1.76 | 2.56 | 0.04 | 1.01 | 0.07 | 3.13 | 0.02 | 0.05 | 0.10 |
| 1.0 | 7500 | -2.30 | -1.56 | -1.21 | -0.77 | -0.49 | -0.22 | 0.01 | 0.27 | 0.55 | 0.89 | 1.31 | 1.66 | 2.33 | 0.04 | 0.98 | 0.02 | 3.00 | 0.01 | 0.04 | 0.09 |
| 1.0 | 10000 | -2.45 | -1.59 | -1.27 | -0.78 | -0.45 | -0.19 | 0.07 | 0.31 | 0.56 | 0.89 | 1.37 | 1.72 | 2.48 | 0.05 | 1.01 | -0.06 | 3.16 | 0.01 | 0.06 | 0.11 |

**Table 7:** **Finite sample behavior of the z_ρ - parameter test concerning the non-observable interactions in the case of r>0 and β2=1**

| | | percentiles | | | | | | | | | | | | | | | | | p-value | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **r** | **T** | **0.01** | **0.05** | **0.1** | **0.2** | **0.3** | **0.4** | **0.5** | **0.6** | **0.7** | **0.8** | **0.9** | **0.95** | **0.99** | **mean** | **std.** | **skew.** | **kurt.** | **0.01** | **0.05** | **0.1** |
| 0.1 | 100 | -4.07 | -2.09 | -1.50 | -0.91 | -0.53 | -0.27 | -0.02 | 0.21 | 0.50 | 0.80 | 1.51 | 2.11 | 4.85 | -0.04 | 1.60 | -1.47 | 23.14 | 0.07 | 0.12 | 0.17 |
| 0.1 | 150 | -4.00 | -1.97 | -1.46 | -0.82 | -0.51 | -0.26 | 0.00 | 0.21 | 0.47 | 0.81 | 1.35 | 1.87 | 3.85 | -0.06 | 1.42 | -2.04 | 22.25 | 0.05 | 0.10 | 0.15 |
| 0.1 | 200 | -2.33 | -1.56 | -1.15 | -0.73 | -0.36 | -0.09 | 0.15 | 0.42 | 0.71 | 1.12 | 1.59 | 2.05 | 3.40 | 0.21 | 1.14 | 0.43 | 4.12 | 0.03 | 0.08 | 0.14 |
| 0.1 | 250 | -2.07 | -1.33 | -1.02 | -0.63 | -0.33 | -0.07 | 0.19 | 0.43 | 0.76 | 1.06 | 1.63 | 2.15 | 3.21 | 0.26 | 1.09 | 0.50 | 3.90 | 0.03 | 0.08 | 0.12 |
| 0.1 | 500 | -2.10 | -1.31 | -1.02 | -0.59 | -0.24 | 0.02 | 0.25 | 0.58 | 0.86 | 1.20 | 1.65 | 2.01 | 2.98 | 0.31 | 1.06 | 0.14 | 3.48 | 0.03 | 0.07 | 0.13 |
| 0.1 | 750 | -2.14 | -1.41 | -0.98 | -0.53 | -0.22 | 0.07 | 0.33 | 0.63 | 0.87 | 1.25 | 1.70 | 2.02 | 2.86 | 0.35 | 1.06 | 0.05 | 3.16 | 0.02 | 0.07 | 0.14 |
| 0.1 | 1000 | -1.91 | -1.21 | -0.85 | -0.43 | -0.14 | 0.16 | 0.41 | 0.70 | 0.94 | 1.26 | 1.72 | 2.13 | 2.83 | 0.42 | 1.00 | 0.06 | 2.99 | 0.01 | 0.07 | 0.13 |
| 0.1 | 1500 | -2.03 | -1.21 | -0.78 | -0.35 | -0.04 | 0.25 | 0.51 | 0.78 | 1.05 | 1.37 | 1.81 | 2.18 | 2.93 | 0.51 | 1.03 | -0.04 | 3.18 | 0.02 | 0.09 | 0.16 |
| 0.1 | 2000 | -1.51 | -1.04 | -0.68 | -0.29 | 0.08 | 0.35 | 0.60 | 0.87 | 1.15 | 1.46 | 1.89 | 2.22 | 2.91 | 0.60 | 0.99 | -0.03 | 2.85 | 0.02 | 0.09 | 0.16 |
| 0.2 | 100 | -3.97 | -1.73 | -1.14 | -0.63 | -0.28 | -0.06 | 0.19 | 0.44 | 0.72 | 1.12 | 1.78 | 2.61 | 5.30 | 0.24 | 1.68 | -2.54 | 35.35 | 0.07 | 0.12 | 0.17 |
| 0.2 | 150 | -2.75 | -1.57 | -1.05 | -0.56 | -0.22 | 0.02 | 0.24 | 0.54 | 0.79 | 1.12 | 1.77 | 2.30 | 4.72 | 0.31 | 1.36 | 1.02 | 14.79 | 0.05 | 0.10 | 0.16 |
| 0.2 | 200 | -2.13 | -1.36 | -1.02 | -0.54 | -0.18 | 0.10 | 0.31 | 0.54 | 0.86 | 1.27 | 1.78 | 2.33 | 3.86 | 0.38 | 1.19 | 0.14 | 8.12 | 0.04 | 0.10 | 0.15 |
| 0.2 | 250 | -1.84 | -1.16 | -0.82 | -0.42 | -0.14 | 0.08 | 0.34 | 0.62 | 0.95 | 1.25 | 1.86 | 2.35 | 3.48 | 0.46 | 1.11 | 0.68 | 4.47 | 0.04 | 0.10 | 0.14 |
| 0.2 | 500 | -1.90 | -1.07 | -0.73 | -0.29 | 0.04 | 0.27 | 0.54 | 0.82 | 1.14 | 1.50 | 1.95 | 2.34 | 3.22 | 0.59 | 1.08 | 0.23 | 3.75 | 0.04 | 0.11 | 0.18 |
| 0.2 | 750 | -1.77 | -0.98 | -0.64 | -0.18 | 0.09 | 0.42 | 0.67 | 0.94 | 1.23 | 1.62 | 2.08 | 2.43 | 3.21 | 0.69 | 1.06 | 0.13 | 3.14 | 0.04 | 0.12 | 0.21 |
| 0.2 | 1000 | -1.58 | -0.81 | -0.48 | -0.02 | 0.26 | 0.55 | 0.82 | 1.05 | 1.34 | 1.67 | 2.13 | 2.55 | 3.22 | 0.82 | 1.01 | 0.07 | 2.97 | 0.05 | 0.13 | 0.21 |
| 0.2 | 1500 | -1.54 | -0.70 | -0.29 | 0.13 | 0.44 | 0.71 | 1.04 | 1.26 | 1.52 | 1.88 | 2.34 | 2.65 | 3.37 | 1.00 | 1.04 | -0.05 | 3.13 | 0.06 | 0.19 | 0.27 |
| 0.2 | 2000 | -1.03 | -0.46 | -0.16 | 0.28 | 0.62 | 0.90 | 1.16 | 1.45 | 1.70 | 2.03 | 2.50 | 2.76 | 3.53 | 1.17 | 1.01 | -0.01 | 2.83 | 0.08 | 0.23 | 0.32 |
| 0.2 | 2500 | -0.85 | -0.29 | 0.00 | 0.44 | 0.75 | 0.99 | 1.29 | 1.52 | 1.79 | 2.12 | 2.57 | 2.92 | 3.79 | 1.28 | 1.00 | 0.15 | 3.03 | 0.10 | 0.25 | 0.35 |
| 0.2 | 5000 | -0.52 | 0.20 | 0.54 | 1.01 | 1.25 | 1.53 | 1.75 | 2.01 | 2.33 | 2.63 | 3.07 | 3.49 | 3.98 | 1.80 | 0.99 | 0.05 | 2.95 | 0.22 | 0.42 | 0.55 |
| 0.2 | 7500 | 0.09 | 0.56 | 0.92 | 1.37 | 1.65 | 1.98 | 2.21 | 2.45 | 2.70 | 3.02 | 3.35 | 3.73 | 4.59 | 2.19 | 0.96 | 0.02 | 2.97 | 0.35 | 0.61 | 0.70 |
| 0.2 | 10000 | 0.06 | 0.89 | 1.25 | 1.67 | 1.99 | 2.26 | 2.54 | 2.78 | 3.05 | 3.35 | 3.81 | 4.07 | 4.86 | 2.52 | 1.00 | 0.02 | 3.13 | 0.49 | 0.71 | 0.81 |
| 0.2 | 15000 | 0.57 | 1.37 | 1.67 | 2.20 | 2.55 | 2.81 | 3.09 | 3.31 | 3.59 | 3.89 | 4.39 | 4.74 | 5.40 | 3.05 | 1.02 | -0.05 | 2.88 | 0.69 | 0.85 | 0.90 |
| 0.2 | 20000 | 1.07 | 1.83 | 2.23 | 2.66 | 2.99 | 3.25 | 3.48 | 3.71 | 4.01 | 4.38 | 4.88 | 5.25 | 5.83 | 3.51 | 1.02 | 0.03 | 2.89 | 0.83 | 0.93 | 0.97 |
| 0.2 | 30000 | 1.27 | 1.87 | 2.26 | 2.68 | 2.99 | 3.28 | 3.51 | 3.76 | 4.06 | 4.39 | 4.80 | 5.02 | 5.58 | 3.51 | 0.97 | -0.07 | 2.74 | 0.83 | 0.94 | 0.97 |
| 0.2 | 40000 | 1.07 | 1.83 | 2.23 | 2.66 | 3.00 | 3.25 | 3.48 | 3.71 | 4.01 | 4.38 | 4.88 | 5.25 | 5.83 | 3.51 | 1.02 | 0.03 | 2.89 | 0.83 | 0.93 | 0.97 |
| 0.3 | 100 | -2.82 | -1.53 | -0.98 | -0.51 | -0.16 | 0.08 | 0.30 | 0.56 | 0.85 | 1.32 | 1.97 | 2.82 | 5.70 | 0.44 | 1.59 | -0.57 | 18.65 | 0.08 | 0.13 | 0.19 |
| 0.3 | 150 | -2.55 | -1.45 | -0.91 | -0.44 | -0.12 | 0.18 | 0.42 | 0.70 | 0.98 | 1.36 | 2.00 | 2.66 | 5.91 | 0.51 | 1.46 | 0.43 | 15.46 | 0.06 | 0.13 | 0.18 |
| 0.3 | 200 | -1.87 | -1.12 | -0.77 | -0.34 | -0.05 | 0.24 | 0.47 | 0.74 | 1.09 | 1.46 | 2.08 | 2.74 | 4.05 | 0.62 | 1.34 | 3.17 | 37.39 | 0.06 | 0.12 | 0.19 |
| 0.3 | 250 | -1.53 | -0.96 | -0.59 | -0.22 | 0.05 | 0.28 | 0.53 | 0.82 | 1.15 | 1.52 | 2.19 | 2.73 | 4.44 | 0.69 | 1.21 | 1.49 | 11.01 | 0.06 | 0.13 | 0.18 |
| 0.3 | 500 | -1.50 | -0.81 | -0.43 | -0.06 | 0.28 | 0.54 | 0.85 | 1.14 | 1.46 | 1.80 | 2.33 | 2.80 | 3.82 | 0.90 | 1.12 | 0.36 | 3.92 | 0.07 | 0.16 | 0.25 |
| 0.3 | 750 | -1.28 | -0.65 | -0.32 | 0.15 | 0.46 | 0.71 | 1.01 | 1.30 | 1.60 | 1.99 | 2.47 | 3.00 | 3.80 | 1.06 | 1.10 | 0.27 | 3.30 | 0.08 | 0.21 | 0.29 |
| 0.3 | 1000 | -1.19 | -0.47 | -0.07 | 0.36 | 0.66 | 0.94 | 1.21 | 1.48 | 1.77 | 2.10 | 2.56 | 2.96 | 3.75 | 1.23 | 1.04 | 0.11 | 2.99 | 0.10 | 0.24 | 0.35 |
| 0.3 | 1500 | -1.04 | -0.19 | 0.19 | 0.60 | 0.93 | 1.24 | 1.50 | 1.76 | 2.03 | 2.43 | 2.83 | 3.29 | 4.14 | 1.51 | 1.07 | 0.02 | 3.16 | 0.16 | 0.33 | 0.45 |
| 0.3 | 2000 | -0.59 | 0.04 | 0.42 | 0.85 | 1.17 | 1.47 | 1.75 | 2.04 | 2.31 | 2.59 | 3.10 | 3.44 | 4.17 | 1.75 | 1.03 | 0.02 | 2.93 | 0.21 | 0.43 | 0.54 |

**Table 7 (cont.): Finite sample behavior of the $z_\rho$ - parameter test concerning the non-observable interactions in the case of r>0 and $\beta$2=1**

| | | percentiles | | | | | | | | | | | | | | | | | p-value | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| r | T | 0.01 | 0.05 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 0.95 | 0.99 | mean | std. | skew. | kurt. | 0.01 | 0.05 | 0.1 |
| 0.4 | 100 | -2.81 | -1.31 | -0.81 | -0.37 | -0.07 | 0.18 | 0.45 | 0.70 | 1.01 | 1.51 | 2.26 | 3.26 | 6.50 | 0.61 | 1.65 | -0.26 | 17.31 | 0.09 | 0.16 | 0.20 |
| 0.4 | 150 | -2.01 | -1.23 | -0.75 | -0.27 | 0.03 | 0.32 | 0.56 | 0.86 | 1.16 | 1.51 | 2.21 | 3.12 | 6.33 | 0.73 | 1.77 | 2.89 | 68.17 | 0.07 | 0.14 | 0.20 |
| 0.4 | 200 | -1.80 | -1.00 | -0.62 | -0.19 | 0.13 | 0.41 | 0.67 | 0.96 | 1.28 | 1.65 | 2.36 | 3.15 | 5.05 | 0.83 | 1.45 | 3.56 | 40.80 | 0.08 | 0.16 | 0.22 |
| 0.4 | 250 | -1.40 | -0.81 | -0.44 | -0.03 | 0.24 | 0.48 | 0.77 | 1.04 | 1.38 | 1.79 | 2.41 | 3.06 | 4.63 | 0.92 | 1.31 | 2.63 | 27.92 | 0.09 | 0.16 | 0.23 |
| 0.4 | 500 | -1.13 | -0.56 | -0.22 | 0.21 | 0.57 | 0.85 | 1.15 | 1.47 | 1.77 | 2.14 | 2.69 | 3.23 | 4.32 | 1.20 | 1.15 | 0.37 | 3.70 | 0.11 | 0.25 | 0.35 |
| 0.4 | 750 | -0.85 | -0.34 | 0.00 | 0.51 | 0.79 | 1.07 | 1.41 | 1.68 | 1.98 | 2.39 | 2.91 | 3.40 | 4.20 | 1.44 | 1.14 | 0.29 | 3.22 | 0.16 | 0.31 | 0.41 |
| 0.4 | 1000 | -0.67 | -0.08 | 0.32 | 0.71 | 1.10 | 1.36 | 1.63 | 1.92 | 2.23 | 2.55 | 3.01 | 3.47 | 4.31 | 1.66 | 1.07 | 0.15 | 3.03 | 0.19 | 0.38 | 0.50 |
| 0.4 | 1500 | -0.69 | 0.29 | 0.60 | 1.09 | 1.44 | 1.75 | 2.01 | 2.26 | 2.60 | 2.95 | 3.43 | 3.86 | 4.77 | 2.02 | 1.10 | 0.04 | 3.09 | 0.31 | 0.53 | 0.62 |
| 0.4 | 2000 | -0.04 | 0.62 | 0.98 | 1.44 | 1.77 | 2.04 | 2.31 | 2.64 | 2.90 | 3.20 | 3.70 | 4.10 | 4.88 | 2.33 | 1.06 | 0.07 | 3.03 | 0.42 | 0.63 | 0.74 |
| 0.5 | 100 | -2.48 | -1.06 | -0.69 | -0.23 | 0.07 | 0.28 | 0.55 | 0.85 | 1.16 | 1.62 | 2.48 | 3.32 | 5.40 | 0.76 | 1.50 | 0.99 | 8.17 | 0.11 | 0.18 | 0.22 |
| 0.5 | 150 | -1.74 | -1.02 | -0.63 | -0.14 | 0.22 | 0.46 | 0.72 | 1.04 | 1.35 | 1.74 | 2.54 | 3.45 | 7.56 | 0.98 | 2.23 | 11.31 | 229.04 | 0.10 | 0.17 | 0.23 |
| 0.5 | 200 | -1.60 | -0.73 | -0.45 | 0.00 | 0.29 | 0.57 | 0.82 | 1.17 | 1.51 | 1.93 | 2.65 | 3.42 | 5.49 | 1.03 | 1.41 | 1.93 | 14.00 | 0.11 | 0.19 | 0.27 |
| 0.5 | 250 | -1.19 | -0.59 | -0.22 | 0.16 | 0.41 | 0.69 | 0.94 | 1.23 | 1.59 | 1.99 | 2.70 | 3.36 | 5.35 | 1.13 | 1.29 | 1.35 | 8.11 | 0.11 | 0.21 | 0.29 |
| 0.5 | 500 | -0.90 | -0.25 | 0.08 | 0.49 | 0.87 | 1.14 | 1.43 | 1.78 | 2.10 | 2.46 | 3.04 | 3.58 | 4.80 | 1.53 | 1.20 | 0.64 | 4.71 | 0.17 | 0.34 | 0.44 |
| 0.5 | 750 | -0.63 | 0.03 | 0.39 | 0.83 | 1.13 | 1.47 | 1.78 | 2.04 | 2.38 | 2.81 | 3.38 | 3.90 | 4.89 | 1.83 | 1.19 | 0.47 | 3.72 | 0.25 | 0.44 | 0.55 |
| 0.5 | 1000 | -0.22 | 0.34 | 0.76 | 1.17 | 1.50 | 1.77 | 2.06 | 2.37 | 2.71 | 3.01 | 3.52 | 4.08 | 4.90 | 2.11 | 1.11 | 0.25 | 3.16 | 0.34 | 0.53 | 0.63 |
| 0.5 | 1500 | -0.25 | 0.78 | 1.11 | 1.59 | 1.90 | 2.24 | 2.54 | 2.82 | 3.16 | 3.56 | 4.04 | 4.54 | 5.53 | 2.57 | 1.15 | 0.16 | 3.11 | 0.48 | 0.69 | 0.79 |
| 0.5 | 2000 | 0.46 | 1.17 | 1.55 | 2.05 | 2.34 | 2.61 | 2.98 | 3.26 | 3.54 | 3.90 | 4.38 | 4.83 | 5.77 | 2.97 | 1.11 | 0.16 | 3.11 | 0.62 | 0.82 | 0.89 |
| 0.5 | 2500 | 0.99 | 1.57 | 1.88 | 2.33 | 2.69 | 2.94 | 3.28 | 3.54 | 3.83 | 4.22 | 4.74 | 5.13 | 6.19 | 3.30 | 1.11 | 0.28 | 3.18 | 0.74 | 0.89 | 0.94 |
| 0.5 | 5000 | 2.20 | 2.88 | 3.27 | 3.71 | 4.08 | 4.34 | 4.59 | 4.91 | 5.21 | 5.59 | 6.11 | 6.53 | 7.31 | 4.65 | 1.10 | 0.12 | 2.91 | 0.98 | 1.00 | 1.00 |
| 0.5 | 7500 | 3.26 | 3.85 | 4.29 | 4.71 | 5.15 | 5.45 | 5.72 | 5.96 | 6.25 | 6.55 | 7.01 | 7.34 | 8.30 | 5.68 | 1.07 | 0.00 | 2.92 | 1.00 | 1.00 | 1.00 |
| 0.5 | 10000 | 4.01 | 4.74 | 5.12 | 5.58 | 5.97 | 6.29 | 6.54 | 6.83 | 7.11 | 7.45 | 7.92 | 8.34 | 9.33 | 6.54 | 1.11 | 0.12 | 3.20 | 1.00 | 1.00 | 1.00 |

**Table 8: Finite sample behavior of the z_β2 - parameter test concerning the non-observable interactions in the case of r≥0 and β2=1**

| | | percentiles | | | | | | | | | | | | | | | | | p-value | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **r** | **T** | **0.01** | **0.05** | **0.1** | **0.2** | **0.3** | **0.4** | **0.5** | **0.6** | **0.7** | **0.8** | **0.9** | **0.95** | **0.99** | **mean** | **std.** | **skew.** | **kurt.** | **0.01** | **0.05** | **0.1** |
| 0.0 | 100 | -2.08 | -0.67 | -0.15 | 0.28 | 0.57 | 0.84 | 1.13 | 1.41 | 1.70 | 2.16 | 3.06 | 3.66 | 5.69 | 1.27 | 1.47 | 0.30 | 8.07 | 0.15 | 0.25 | 0.33 |
| 0.0 | 150 | -1.28 | -0.21 | 0.16 | 0.60 | 0.89 | 1.17 | 1.40 | 1.67 | 2.04 | 2.48 | 3.15 | 3.93 | 5.80 | 1.59 | 1.39 | 1.59 | 10.10 | 0.19 | 0.32 | 0.41 |
| 0.0 | 200 | -0.73 | -0.04 | 0.31 | 0.76 | 1.05 | 1.30 | 1.61 | 1.90 | 2.21 | 2.63 | 3.36 | 3.99 | 5.21 | 1.73 | 1.24 | 0.78 | 4.84 | 0.21 | 0.37 | 0.48 |
| 0.0 | 250 | -0.48 | 0.03 | 0.50 | 0.99 | 1.33 | 1.58 | 1.82 | 2.06 | 2.31 | 2.75 | 3.24 | 3.73 | 4.89 | 1.86 | 1.09 | 0.32 | 3.48 | 0.24 | 0.45 | 0.58 |
| 0.0 | 500 | 0.07 | 0.88 | 1.23 | 1.73 | 2.05 | 2.34 | 2.57 | 2.83 | 3.15 | 3.49 | 3.99 | 4.56 | 5.35 | 2.62 | 1.09 | 0.30 | 3.58 | 0.50 | 0.72 | 0.83 |
| 0.0 | 750 | 0.94 | 1.57 | 1.85 | 2.32 | 2.60 | 2.89 | 3.19 | 3.50 | 3.79 | 4.20 | 4.69 | 5.25 | 6.13 | 3.25 | 1.13 | 0.37 | 3.24 | 0.71 | 0.88 | 0.94 |
| 0.0 | 1000 | 1.40 | 2.07 | 2.44 | 2.85 | 3.15 | 3.39 | 3.65 | 3.94 | 4.27 | 4.63 | 5.13 | 5.63 | 6.39 | 3.73 | 1.06 | 0.23 | 2.93 | 0.86 | 0.96 | 0.98 |
| 0.0 | 1500 | 2.18 | 2.88 | 3.15 | 3.61 | 3.89 | 4.26 | 4.51 | 4.75 | 5.12 | 5.44 | 6.00 | 6.45 | 7.44 | 4.55 | 1.11 | 0.34 | 3.28 | 0.97 | 1.00 | 1.00 |
| 0.0 | 2000 | 2.95 | 3.55 | 3.94 | 4.38 | 4.70 | 4.93 | 5.18 | 5.48 | 5.77 | 6.11 | 6.60 | 7.05 | 7.80 | 5.24 | 1.04 | 0.18 | 3.12 | 0.99 | 1.00 | 1.00 |
| 0.0 | 2500 | 3.39 | 4.15 | 4.61 | 5.00 | 5.31 | 5.59 | 5.85 | 6.16 | 6.43 | 6.79 | 7.23 | 7.72 | 8.40 | 5.89 | 1.06 | 0.06 | 3.04 | 1.00 | 1.00 | 1.00 |
| 0.0 | 5000 | 6.02 | 6.64 | 6.97 | 7.37 | 7.71 | 8.02 | 8.30 | 8.58 | 8.83 | 9.19 | 9.67 | 10.07 | 10.73 | 8.30 | 1.05 | 0.09 | 2.75 | 1.00 | 1.00 | 1.00 |
| 0.0 | 7500 | 7.86 | 8.52 | 8.89 | 9.32 | 9.63 | 9.86 | 10.12 | 10.41 | 10.73 | 11.07 | 11.51 | 11.89 | 12.50 | 10.17 | 1.02 | 0.05 | 2.79 | 1.00 | 1.00 | 1.00 |
| 0.0 | 10000 | 9.20 | 10.09 | 10.43 | 10.85 | 11.16 | 11.45 | 11.72 | 11.99 | 12.26 | 12.61 | 13.06 | 13.48 | 14.49 | 11.74 | 1.05 | 0.08 | 3.16 | 1.00 | 1.00 | 1.00 |
| 0.1 | 100 | -2.08 | -0.67 | -0.15 | 0.28 | 0.57 | 0.84 | 1.13 | 1.41 | 1.70 | 2.16 | 3.06 | 3.66 | 5.69 | 1.27 | 1.47 | 0.30 | 8.07 | 0.15 | 0.25 | 0.33 |
| 0.1 | 150 | -1.28 | -0.21 | 0.16 | 0.60 | 0.89 | 1.17 | 1.40 | 1.67 | 2.04 | 2.48 | 3.15 | 3.93 | 5.80 | 1.59 | 1.39 | 1.59 | 10.10 | 0.19 | 0.32 | 0.41 |
| 0.1 | 200 | -0.78 | -0.08 | 0.31 | 0.78 | 1.03 | 1.30 | 1.59 | 1.89 | 2.19 | 2.61 | 3.28 | 3.85 | 4.86 | 1.70 | 1.18 | 0.53 | 3.77 | 0.21 | 0.38 | 0.48 |
| 0.1 | 250 | -0.42 | 0.07 | 0.46 | 0.98 | 1.29 | 1.58 | 1.80 | 2.03 | 2.30 | 2.71 | 3.19 | 3.68 | 4.89 | 1.84 | 1.08 | 0.30 | 3.57 | 0.24 | 0.43 | 0.57 |
| 0.1 | 500 | 0.12 | 0.95 | 1.23 | 1.71 | 2.05 | 2.30 | 2.54 | 2.76 | 3.14 | 3.50 | 3.97 | 4.46 | 5.21 | 2.60 | 1.08 | 0.31 | 3.67 | 0.48 | 0.72 | 0.82 |
| 0.1 | 750 | 0.88 | 1.52 | 1.84 | 2.29 | 2.57 | 2.86 | 3.16 | 3.46 | 3.75 | 4.16 | 4.62 | 5.16 | 6.20 | 3.22 | 1.12 | 0.39 | 3.41 | 0.69 | 0.87 | 0.94 |
| 0.1 | 1000 | 1.38 | 2.09 | 2.41 | 2.83 | 3.12 | 3.35 | 3.62 | 3.91 | 4.20 | 4.59 | 5.11 | 5.51 | 6.20 | 3.70 | 1.04 | 0.24 | 2.93 | 0.86 | 0.96 | 0.98 |
| 0.1 | 1500 | 2.19 | 2.86 | 3.11 | 3.55 | 3.92 | 4.23 | 4.45 | 4.72 | 5.05 | 5.37 | 5.95 | 6.30 | 7.30 | 4.51 | 1.08 | 0.31 | 3.23 | 0.97 | 1.00 | 1.00 |
| 0.1 | 2000 | 2.93 | 3.57 | 3.91 | 4.32 | 4.65 | 4.90 | 5.15 | 5.42 | 5.73 | 6.10 | 6.51 | 6.94 | 7.61 | 5.19 | 1.03 | 0.17 | 3.14 | 1.00 | 1.00 | 1.00 |
| 0.2 | 100 | -1.93 | -0.62 | -0.14 | 0.28 | 0.59 | 0.85 | 1.09 | 1.37 | 1.63 | 2.07 | 2.78 | 3.45 | 5.08 | 1.21 | 1.28 | 0.32 | 4.98 | 0.13 | 0.24 | 0.32 |
| 0.2 | 150 | -1.25 | -0.21 | 0.16 | 0.59 | 0.89 | 1.18 | 1.39 | 1.65 | 1.95 | 2.47 | 3.01 | 3.70 | 5.52 | 1.53 | 1.30 | 0.87 | 9.44 | 0.18 | 0.30 | 0.41 |
| 0.2 | 200 | -0.80 | -0.06 | 0.31 | 0.79 | 1.06 | 1.30 | 1.60 | 1.87 | 2.14 | 2.59 | 3.23 | 3.77 | 4.96 | 1.70 | 1.23 | 1.83 | 19.71 | 0.20 | 0.37 | 0.49 |
| 0.2 | 250 | -0.47 | 0.08 | 0.50 | 1.01 | 1.34 | 1.59 | 1.81 | 2.03 | 2.27 | 2.66 | 3.16 | 3.60 | 4.58 | 1.83 | 1.06 | 0.25 | 3.54 | 0.22 | 0.43 | 0.57 |
| 0.2 | 500 | 0.10 | 0.95 | 1.23 | 1.73 | 2.02 | 2.31 | 2.56 | 2.75 | 3.14 | 3.46 | 3.93 | 4.33 | 5.13 | 2.58 | 1.06 | 0.29 | 3.78 | 0.49 | 0.72 | 0.82 |
| 0.2 | 750 | 0.85 | 1.51 | 1.84 | 2.30 | 2.56 | 2.86 | 3.14 | 3.45 | 3.73 | 4.12 | 4.55 | 5.02 | 6.13 | 3.19 | 1.09 | 0.35 | 3.32 | 0.70 | 0.87 | 0.93 |
| 0.2 | 1000 | 1.42 | 2.08 | 2.42 | 2.79 | 3.10 | 3.35 | 3.60 | 3.86 | 4.16 | 4.53 | 5.01 | 5.45 | 6.16 | 3.67 | 1.02 | 0.25 | 2.91 | 0.86 | 0.97 | 0.98 |
| 0.2 | 1500 | 2.14 | 2.82 | 3.13 | 3.52 | 3.90 | 4.19 | 4.43 | 4.68 | 4.99 | 5.32 | 5.84 | 6.23 | 7.12 | 4.46 | 1.05 | 0.29 | 3.20 | 0.97 | 1.00 | 1.00 |
| 0.2 | 2000 | 2.86 | 3.51 | 3.89 | 4.26 | 4.65 | 4.87 | 5.09 | 5.34 | 5.66 | 6.00 | 6.49 | 6.82 | 7.42 | 5.15 | 1.00 | 0.15 | 3.07 | 1.00 | 1.00 | 1.00 |
| 0.2 | 2500 | 3.40 | 4.15 | 4.48 | 4.94 | 5.25 | 5.48 | 5.76 | 6.03 | 6.30 | 6.60 | 7.06 | 7.48 | 8.21 | 5.78 | 1.01 | 0.06 | 3.06 | 1.00 | 1.00 | 1.00 |
| 0.2 | 5000 | 5.96 | 6.54 | 6.84 | 7.30 | 7.61 | 7.88 | 8.14 | 8.42 | 8.68 | 9.02 | 9.48 | 9.79 | 10.58 | 8.16 | 1.00 | 0.07 | 2.77 | 1.00 | 1.00 | 1.00 |
| 0.2 | 7500 | 7.74 | 8.39 | 8.75 | 9.19 | 9.48 | 9.73 | 9.94 | 10.22 | 10.51 | 10.83 | 11.29 | 11.62 | 12.27 | 9.99 | 0.97 | 0.04 | 2.82 | 1.00 | 1.00 | 1.00 |
| 0.2 | 10000 | 9.17 | 9.99 | 10.30 | 10.69 | 11.00 | 11.27 | 11.54 | 11.77 | 12.05 | 12.35 | 12.80 | 13.22 | 14.10 | 11.53 | 1.00 | 0.09 | 3.20 | 1.00 | 1.00 | 1.00 |
| 0.2 | 15000 | 11.79 | 12.47 | 12.80 | 13.25 | 13.57 | 13.87 | 14.12 | 14.39 | 14.67 | 15.02 | 15.56 | 15.93 | 16.46 | 14.15 | 1.04 | 0.07 | 2.73 | 1.00 | 1.00 | 1.00 |
| 0.2 | 20000 | 14.05 | 14.71 | 14.98 | 15.45 | 15.79 | 16.06 | 16.34 | 16.63 | 16.87 | 17.18 | 17.71 | 18.05 | 18.80 | 16.34 | 1.02 | 0.07 | 2.78 | 1.00 | 1.00 | 1.00 |
| 0.2 | 30000 | 14.19 | 14.79 | 15.09 | 15.50 | 15.80 | 16.05 | 16.27 | 16.57 | 16.90 | 17.21 | 17.64 | 18.00 | 18.68 | 16.35 | 0.99 | 0.16 | 3.02 | 1.00 | 1.00 | 1.00 |
| 0.2 | 40000 | 14.05 | 14.71 | 14.98 | 15.45 | 15.79 | 16.06 | 16.34 | 16.63 | 16.87 | 17.18 | 17.71 | 18.06 | 18.80 | 16.35 | 1.05 | 0.48 | 5.75 | 1.00 | 1.00 | 1.00 |

**Table 8 (cont.): Finite sample behavior of the z_β2 - parameter test concerning the non-observable interactions in the case of r≥0 and β2=1**

| | | percentiles | | | | | | | | | | | | | | | | | p-value | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **r** | **T** | **0.01** | **0.05** | **0.1** | **0.2** | **0.3** | **0.4** | **0.5** | **0.6** | **0.7** | **0.8** | **0.9** | **0.95** | **0.99** | **mean** | **std.** | **skew.** | **kurt.** | **0.01** | **0.05** | **0.1** |
| 0.3 | 100 | -2.39 | -0.64 | -0.13 | 0.28 | 0.60 | 0.83 | 1.11 | 1.37 | 1.67 | 2.01 | 2.69 | 3.42 | 4.69 | 1.18 | 1.27 | -0.06 | 6.11 | 0.12 | 0.22 | 0.32 |
| 0.3 | 150 | -1.24 | -0.20 | 0.11 | 0.59 | 0.87 | 1.14 | 1.38 | 1.64 | 1.92 | 2.44 | 2.94 | 3.69 | 4.95 | 1.49 | 1.26 | 0.64 | 10.01 | 0.17 | 0.29 | 0.40 |
| 0.3 | 200 | -0.70 | -0.05 | 0.30 | 0.78 | 1.06 | 1.28 | 1.56 | 1.86 | 2.14 | 2.54 | 3.13 | 3.71 | 4.66 | 1.66 | 1.14 | 0.31 | 4.16 | 0.19 | 0.36 | 0.48 |
| 0.3 | 250 | -0.51 | 0.07 | 0.52 | 1.00 | 1.30 | 1.59 | 1.79 | 2.01 | 2.24 | 2.64 | 3.10 | 3.57 | 4.60 | 1.81 | 1.04 | 0.15 | 3.83 | 0.21 | 0.43 | 0.57 |
| 0.3 | 500 | 0.11 | 0.90 | 1.22 | 1.70 | 1.99 | 2.31 | 2.51 | 2.77 | 3.08 | 3.40 | 3.88 | 4.27 | 5.14 | 2.56 | 1.05 | 0.27 | 3.84 | 0.47 | 0.71 | 0.82 |
| 0.3 | 750 | 0.93 | 1.56 | 1.83 | 2.27 | 2.56 | 2.83 | 3.10 | 3.40 | 3.68 | 4.04 | 4.54 | 5.02 | 5.86 | 3.16 | 1.07 | 0.32 | 3.32 | 0.70 | 0.88 | 0.94 |
| 0.3 | 1000 | 1.39 | 2.12 | 2.42 | 2.78 | 3.09 | 3.31 | 3.56 | 3.81 | 4.13 | 4.47 | 4.98 | 5.34 | 6.13 | 3.64 | 1.00 | 0.25 | 2.92 | 0.87 | 0.96 | 0.98 |
| 0.3 | 1500 | 2.10 | 2.81 | 3.13 | 3.51 | 3.88 | 4.16 | 4.38 | 4.66 | 4.97 | 5.27 | 5.81 | 6.09 | 7.09 | 4.43 | 1.03 | 0.25 | 3.22 | 0.97 | 1.00 | 1.00 |
| 0.3 | 2000 | 2.91 | 3.49 | 3.88 | 4.26 | 4.61 | 4.86 | 5.05 | 5.32 | 5.61 | 5.96 | 6.40 | 6.74 | 7.36 | 5.11 | 0.98 | 0.14 | 3.03 | 1.00 | 1.00 | 1.00 |
| 0.4 | 100 | -1.93 | -0.59 | -0.14 | 0.27 | 0.60 | 0.82 | 1.10 | 1.34 | 1.61 | 1.96 | 2.58 | 3.32 | 4.68 | 1.16 | 1.26 | -0.04 | 6.31 | 0.11 | 0.21 | 0.30 |
| 0.4 | 150 | -1.17 | -0.19 | 0.15 | 0.57 | 0.86 | 1.13 | 1.37 | 1.62 | 1.90 | 2.33 | 2.92 | 3.49 | 4.56 | 1.45 | 1.17 | -0.08 | 5.84 | 0.17 | 0.28 | 0.39 |
| 0.4 | 200 | -0.79 | -0.07 | 0.30 | 0.79 | 1.04 | 1.30 | 1.58 | 1.81 | 2.10 | 2.53 | 3.11 | 3.51 | 4.64 | 1.64 | 1.13 | 0.05 | 5.61 | 0.19 | 0.35 | 0.47 |
| 0.4 | 250 | -0.41 | 0.08 | 0.48 | 1.00 | 1.29 | 1.56 | 1.78 | 1.99 | 2.23 | 2.57 | 3.08 | 3.55 | 4.65 | 1.79 | 1.03 | 0.08 | 4.27 | 0.20 | 0.42 | 0.57 |
| 0.4 | 500 | 0.09 | 0.94 | 1.21 | 1.70 | 1.99 | 2.27 | 2.51 | 2.74 | 3.04 | 3.36 | 3.88 | 4.28 | 5.10 | 2.54 | 1.03 | 0.28 | 3.78 | 0.47 | 0.71 | 0.81 |
| 0.4 | 750 | 0.97 | 1.56 | 1.84 | 2.25 | 2.55 | 2.83 | 3.09 | 3.40 | 3.65 | 4.01 | 4.50 | 4.95 | 5.65 | 3.14 | 1.05 | 0.29 | 3.21 | 0.69 | 0.87 | 0.93 |
| 0.4 | 1000 | 1.37 | 2.13 | 2.37 | 2.79 | 3.06 | 3.29 | 3.54 | 3.81 | 4.10 | 4.44 | 4.95 | 5.32 | 6.05 | 3.62 | 0.98 | 0.24 | 2.97 | 0.86 | 0.97 | 0.98 |
| 0.4 | 1500 | 2.15 | 2.79 | 3.16 | 3.51 | 3.86 | 4.13 | 4.40 | 4.64 | 4.93 | 5.20 | 5.74 | 6.10 | 6.91 | 4.41 | 1.02 | 0.23 | 3.21 | 0.97 | 0.99 | 1.00 |
| 0.4 | 2000 | 2.89 | 3.50 | 3.89 | 4.30 | 4.58 | 4.82 | 5.04 | 5.30 | 5.57 | 5.89 | 6.32 | 6.66 | 7.35 | 5.09 | 0.96 | 0.14 | 3.13 | 1.00 | 1.00 | 1.00 |
| 0.5 | 100 | -3.00 | -0.62 | -0.08 | 0.26 | 0.60 | 0.82 | 1.08 | 1.33 | 1.58 | 1.90 | 2.51 | 3.07 | 4.10 | 1.10 | 1.21 | -0.72 | 7.76 | 0.11 | 0.20 | 0.29 |
| 0.5 | 150 | -1.19 | -0.15 | 0.16 | 0.60 | 0.87 | 1.14 | 1.34 | 1.59 | 1.89 | 2.31 | 2.85 | 3.45 | 4.51 | 1.44 | 1.18 | 0.32 | 9.53 | 0.16 | 0.28 | 0.38 |
| 0.5 | 200 | -0.90 | -0.07 | 0.33 | 0.77 | 1.05 | 1.30 | 1.59 | 1.82 | 2.09 | 2.47 | 3.03 | 3.48 | 4.53 | 1.61 | 1.10 | -0.10 | 5.92 | 0.19 | 0.35 | 0.48 |
| 0.5 | 250 | -0.45 | 0.12 | 0.53 | 1.01 | 1.31 | 1.56 | 1.79 | 1.99 | 2.21 | 2.56 | 3.00 | 3.41 | 4.19 | 1.78 | 1.00 | -0.05 | 4.47 | 0.20 | 0.42 | 0.55 |
| 0.5 | 500 | 0.13 | 0.98 | 1.26 | 1.70 | 2.03 | 2.27 | 2.52 | 2.69 | 2.97 | 3.31 | 3.82 | 4.14 | 4.98 | 2.52 | 1.00 | 0.19 | 3.75 | 0.46 | 0.73 | 0.81 |
| 0.5 | 750 | 0.88 | 1.55 | 1.87 | 2.28 | 2.52 | 2.81 | 3.07 | 3.37 | 3.66 | 3.96 | 4.39 | 4.85 | 5.61 | 3.12 | 1.02 | 0.26 | 3.28 | 0.69 | 0.88 | 0.94 |
| 0.5 | 1000 | 1.40 | 2.13 | 2.42 | 2.80 | 3.03 | 3.29 | 3.52 | 3.78 | 4.08 | 4.42 | 4.86 | 5.24 | 5.98 | 3.59 | 0.96 | 0.23 | 3.06 | 0.86 | 0.97 | 0.98 |
| 0.5 | 1500 | 2.05 | 2.77 | 3.16 | 3.50 | 3.86 | 4.11 | 4.40 | 4.63 | 4.88 | 5.15 | 5.71 | 6.08 | 6.94 | 4.39 | 1.00 | 0.19 | 3.13 | 0.97 | 0.99 | 1.00 |
| 0.5 | 2000 | 2.78 | 3.55 | 3.87 | 4.28 | 4.56 | 4.77 | 5.04 | 5.29 | 5.56 | 5.87 | 6.29 | 6.63 | 7.32 | 5.06 | 0.95 | 0.12 | 3.15 | 1.00 | 1.00 | 1.00 |
| 0.5 | 2500 | 3.45 | 4.11 | 4.43 | 4.89 | 5.22 | 5.43 | 5.66 | 5.92 | 6.17 | 6.46 | 7.00 | 7.35 | 8.02 | 5.69 | 0.97 | 0.04 | 3.06 | 1.00 | 1.00 | 1.00 |
| 0.5 | 5000 | 6.03 | 6.46 | 6.76 | 7.18 | 7.49 | 7.77 | 8.02 | 8.29 | 8.51 | 8.87 | 9.23 | 9.60 | 10.26 | 8.02 | 0.96 | 0.06 | 2.77 | 1.00 | 1.00 | 1.00 |
| 0.5 | 7500 | 7.74 | 8.33 | 8.67 | 9.05 | 9.31 | 9.58 | 9.80 | 10.04 | 10.31 | 10.61 | 11.09 | 11.36 | 11.98 | 9.83 | 0.92 | 0.02 | 2.86 | 1.00 | 1.00 | 1.00 |
| 0.5 | 10000 | 9.13 | 9.82 | 10.15 | 10.55 | 10.84 | 11.09 | 11.35 | 11.58 | 11.81 | 12.15 | 12.60 | 12.96 | 13.70 | 11.35 | 0.95 | 0.10 | 3.19 | 1.00 | 1.00 | 1.00 |

**Table 9:** **Finite sample behavior of the z_β2 - parameter test concerning the non-observable interactions in the case of r≥0 and β2=1 and additional explanatory variables**

| | | percentiles | | | | | | | | | | | | | | | | p-value | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| r | T | 0.01 | 0.05 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 0.95 | 0.99 | mean | std. | skew. | kurt. | 0.01 | 0.05 | 0.1 |
| 0.0 | 100 | -0.32 | 0.20 | 0.54 | 1.00 | 1.33 | 1.54 | 1.79 | 2.04 | 2.37 | 2.78 | 3.34 | 4.12 | 6.06 | 1.94 | 1.21 | 0.91 | 4.89 | 0.25 | 0.43 | 0.56 |
| 0.0 | 150 | 0.28 | 0.77 | 1.11 | 1.47 | 1.79 | 2.03 | 2.32 | 2.54 | 2.86 | 3.26 | 3.91 | 4.68 | 6.46 | 2.44 | 1.19 | 1.08 | 5.69 | 0.39 | 0.63 | 0.76 |
| 0.0 | 200 | 0.61 | 1.15 | 1.51 | 1.86 | 2.16 | 2.43 | 2.70 | 2.97 | 3.29 | 3.70 | 4.22 | 4.86 | 6.36 | 2.81 | 1.17 | 0.78 | 4.67 | 0.54 | 0.77 | 0.87 |
| 0.0 | 250 | 0.90 | 1.57 | 1.92 | 2.23 | 2.51 | 2.79 | 3.05 | 3.29 | 3.57 | 4.00 | 4.50 | 5.04 | 6.17 | 3.13 | 1.08 | 0.65 | 4.45 | 0.68 | 0.89 | 0.94 |
| 0.0 | 500 | 2.25 | 2.73 | 3.15 | 3.59 | 3.94 | 4.18 | 4.46 | 4.72 | 4.97 | 5.30 | 5.81 | 6.23 | 7.03 | 4.47 | 1.04 | 0.16 | 3.23 | 0.96 | 0.99 | 1.00 |
| 0.0 | 750 | 3.28 | 3.88 | 4.18 | 4.64 | 5.00 | 5.24 | 5.49 | 5.74 | 6.05 | 6.38 | 6.90 | 7.29 | 8.18 | 5.53 | 1.04 | 0.22 | 3.14 | 1.00 | 1.00 | 1.00 |
| 0.0 | 1000 | 4.04 | 4.72 | 5.10 | 5.60 | 5.90 | 6.13 | 6.38 | 6.63 | 6.92 | 7.22 | 7.68 | 8.18 | 8.91 | 6.40 | 1.02 | 0.09 | 3.22 | 1.00 | 1.00 | 1.00 |
| 0.0 | 1500 | 5.40 | 6.01 | 6.50 | 6.96 | 7.26 | 7.55 | 7.76 | 8.06 | 8.34 | 8.65 | 9.17 | 9.59 | 10.28 | 7.81 | 1.05 | 0.11 | 3.12 | 1.00 | 1.00 | 1.00 |
| 0.0 | 2000 | 6.63 | 7.43 | 7.77 | 8.22 | 8.57 | 8.82 | 9.04 | 9.26 | 9.55 | 9.95 | 10.40 | 10.78 | 11.35 | 9.07 | 1.02 | 0.05 | 3.15 | 1.00 | 1.00 | 1.00 |
| 0.0 | 2500 | 7.72 | 8.38 | 8.75 | 9.23 | 9.57 | 9.87 | 10.13 | 10.39 | 10.71 | 11.10 | 11.53 | 11.86 | 12.61 | 10.14 | 1.06 | 0.06 | 2.81 | 1.00 | 1.00 | 1.00 |
| 0.0 | 5000 | 11.98 | 12.70 | 13.03 | 13.43 | 13.74 | 14.05 | 14.33 | 14.59 | 14.91 | 15.23 | 15.84 | 16.15 | 16.69 | 14.35 | 1.07 | 0.05 | 2.97 | 1.00 | 1.00 | 1.00 |
| 0.0 | 7500 | 15.32 | 15.98 | 16.27 | 16.70 | 17.02 | 17.30 | 17.51 | 17.77 | 18.09 | 18.43 | 18.91 | 19.26 | 20.33 | 17.57 | 1.03 | 0.24 | 3.29 | 1.00 | 1.00 | 1.00 |
| 0.0 | 10000 | 17.85 | 18.54 | 18.91 | 19.42 | 19.71 | 19.96 | 20.23 | 20.53 | 20.80 | 21.24 | 21.70 | 22.11 | 22.86 | 20.28 | 1.08 | 0.08 | 2.81 | 1.00 | 1.00 | 1.00 |
| 0.1 | 100 | -0.32 | 0.20 | 0.54 | 1.00 | 1.33 | 1.54 | 1.79 | 2.04 | 2.37 | 2.78 | 3.34 | 4.12 | 6.06 | 1.94 | 1.21 | 0.91 | 4.89 | 0.25 | 0.43 | 0.56 |
| 0.1 | 150 | 0.28 | 0.77 | 1.11 | 1.47 | 1.79 | 2.03 | 2.32 | 2.54 | 2.86 | 3.26 | 3.91 | 4.68 | 6.46 | 2.44 | 1.19 | 1.08 | 5.69 | 0.39 | 0.63 | 0.76 |
| 0.1 | 200 | 0.55 | 1.13 | 1.44 | 1.84 | 2.16 | 2.42 | 2.68 | 2.96 | 3.27 | 3.69 | 4.22 | 4.92 | 6.25 | 2.80 | 1.17 | 0.75 | 4.59 | 0.53 | 0.76 | 0.86 |
| 0.1 | 250 | 0.77 | 1.53 | 1.86 | 2.22 | 2.48 | 2.78 | 3.00 | 3.30 | 3.58 | 3.98 | 4.51 | 4.97 | 6.18 | 3.12 | 1.10 | 0.76 | 5.09 | 0.67 | 0.89 | 0.94 |
| 0.1 | 500 | 2.11 | 2.71 | 3.11 | 3.60 | 3.91 | 4.14 | 4.43 | 4.67 | 4.96 | 5.28 | 5.83 | 6.28 | 7.18 | 4.45 | 1.06 | 0.21 | 3.34 | 0.97 | 0.99 | 1.00 |
| 0.1 | 750 | 3.26 | 3.85 | 4.17 | 4.59 | 4.95 | 5.21 | 5.48 | 5.73 | 6.04 | 6.38 | 6.88 | 7.24 | 8.32 | 5.51 | 1.06 | 0.23 | 3.10 | 1.00 | 1.00 | 1.00 |
| 0.1 | 1000 | 3.95 | 4.63 | 5.06 | 5.55 | 5.84 | 6.09 | 6.37 | 6.64 | 6.91 | 7.21 | 7.68 | 8.16 | 8.91 | 6.38 | 1.04 | 0.09 | 3.25 | 1.00 | 1.00 | 1.00 |
| 0.1 | 1500 | 5.35 | 5.97 | 6.45 | 6.93 | 7.22 | 7.48 | 7.75 | 8.04 | 8.31 | 8.63 | 9.15 | 9.55 | 10.20 | 7.77 | 1.05 | 0.09 | 3.05 | 1.00 | 1.00 | 1.00 |
| 0.1 | 2000 | 6.56 | 7.40 | 7.72 | 8.18 | 8.53 | 8.78 | 8.99 | 9.23 | 9.51 | 9.90 | 10.36 | 10.76 | 11.40 | 9.02 | 1.02 | 0.04 | 3.13 | 1.00 | 1.00 | 1.00 |
| 0.2 | 100 | -0.55 | 0.18 | 0.52 | 0.97 | 1.27 | 1.50 | 1.76 | 2.04 | 2.38 | 2.77 | 3.41 | 4.12 | 5.62 | 1.91 | 1.21 | 0.64 | 4.16 | 0.24 | 0.42 | 0.54 |
| 0.2 | 150 | 0.13 | 0.73 | 1.06 | 1.43 | 1.76 | 2.02 | 2.27 | 2.52 | 2.83 | 3.22 | 3.98 | 4.66 | 6.03 | 2.41 | 1.21 | 0.96 | 5.09 | 0.38 | 0.63 | 0.74 |
| 0.2 | 200 | 0.36 | 1.09 | 1.44 | 1.81 | 2.14 | 2.40 | 2.63 | 2.94 | 3.29 | 3.68 | 4.24 | 4.90 | 6.27 | 2.78 | 1.20 | 0.83 | 5.11 | 0.53 | 0.75 | 0.84 |
| 0.2 | 250 | 0.88 | 1.52 | 1.88 | 2.18 | 2.46 | 2.72 | 3.01 | 3.26 | 3.55 | 3.95 | 4.53 | 5.01 | 6.26 | 3.10 | 1.11 | 0.68 | 4.36 | 0.66 | 0.87 | 0.93 |
| 0.2 | 500 | 2.07 | 2.68 | 3.04 | 3.58 | 3.86 | 4.14 | 4.38 | 4.65 | 4.92 | 5.27 | 5.79 | 6.23 | 7.38 | 4.41 | 1.06 | 0.22 | 3.30 | 0.96 | 0.99 | 1.00 |
| 0.2 | 750 | 3.29 | 3.85 | 4.15 | 4.53 | 4.89 | 5.13 | 5.44 | 5.68 | 5.99 | 6.30 | 6.84 | 7.23 | 8.22 | 5.46 | 1.06 | 0.29 | 3.21 | 1.00 | 1.00 | 1.00 |
| 0.2 | 1000 | 3.90 | 4.62 | 5.01 | 5.46 | 5.77 | 6.04 | 6.29 | 6.57 | 6.87 | 7.17 | 7.58 | 8.03 | 9.11 | 6.32 | 1.04 | 0.11 | 3.20 | 1.00 | 1.00 | 1.00 |
| 0.2 | 1500 | 5.22 | 5.99 | 6.33 | 6.82 | 7.16 | 7.42 | 7.67 | 7.92 | 8.20 | 8.56 | 9.07 | 9.42 | 10.31 | 7.69 | 1.05 | 0.11 | 3.09 | 1.00 | 1.00 | 1.00 |
| 0.2 | 2000 | 6.44 | 7.29 | 7.63 | 8.11 | 8.41 | 8.68 | 8.93 | 9.14 | 9.41 | 9.77 | 10.24 | 10.67 | 11.34 | 8.93 | 1.02 | 0.03 | 3.09 | 1.00 | 1.00 | 1.00 |
| 0.2 | 2500 | 7.57 | 8.22 | 8.54 | 9.06 | 9.40 | 9.73 | 10.00 | 10.25 | 10.56 | 10.90 | 11.37 | 11.72 | 12.50 | 9.99 | 1.07 | 0.06 | 2.95 | 1.00 | 1.00 | 1.00 |
| 0.2 | 5000 | 11.67 | 12.40 | 12.74 | 13.17 | 13.51 | 13.81 | 14.10 | 14.38 | 14.69 | 15.05 | 15.53 | 15.95 | 16.54 | 14.11 | 1.08 | 0.04 | 2.94 | 1.00 | 1.00 | 1.00 |
| 0.2 | 7500 | 14.96 | 15.61 | 15.97 | 16.43 | 16.73 | 17.01 | 17.24 | 17.48 | 17.82 | 18.16 | 18.61 | 19.05 | 19.96 | 17.29 | 1.05 | 0.24 | 3.31 | 1.00 | 1.00 | 1.00 |
| 0.2 | 10000 | 17.52 | 18.19 | 18.58 | 19.00 | 19.35 | 19.62 | 19.90 | 20.20 | 20.50 | 20.94 | 21.41 | 21.80 | 22.57 | 19.95 | 1.10 | 0.11 | 2.80 | 1.00 | 1.00 | 1.00 |
| 0.2 | 15000 | 22.10 | 22.67 | 23.13 | 23.57 | 23.88 | 24.15 | 24.44 | 24.70 | 25.06 | 25.43 | 25.89 | 26.21 | 26.81 | 24.46 | 1.08 | 0.03 | 3.12 | 1.00 | 1.00 | 1.00 |
| 0.2 | 20000 | 25.92 | 26.54 | 26.91 | 27.39 | 27.72 | 28.06 | 28.29 | 28.50 | 28.78 | 29.17 | 29.65 | 29.98 | 30.60 | 28.27 | 1.04 | 0.04 | 2.93 | 1.00 | 1.00 | 1.00 |
| 0.2 | 30000 | 26.08 | 26.58 | 26.89 | 27.35 | 27.69 | 28.02 | 28.29 | 28.53 | 28.83 | 29.15 | 29.63 | 30.05 | 30.85 | 28.28 | 1.07 | 0.41 | 4.36 | 1.00 | 1.00 | 1.00 |
| 0.2 | 40000 | 25.92 | 26.54 | 26.91 | 27.39 | 27.72 | 28.06 | 28.29 | 28.50 | 28.78 | 29.17 | 29.68 | 29.99 | 30.71 | 28.29 | 1.12 | 1.52 | 19.64 | 1.00 | 1.00 | 1.00 |

**Table 9 (cont.): Finite sample behavior of the z_β2 - parameter test concerning the non-observable interactions in the case of r≥0 and β2=1 and additional explanatory variables**

| | | percentiles | | | | | | | | | | | | | | | | | p-value | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| r | T | 0.01 | 0.05 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 0.95 | 0.99 | mean | std. | skew. | kurt. | 0.01 | 0.05 | 0.1 |
| 0.3 | 100 | -0.45 | 0.14 | 0.47 | 0.93 | 1.23 | 1.49 | 1.73 | 2.00 | 2.34 | 2.77 | 3.50 | 4.21 | 5.62 | 1.89 | 1.23 | 0.69 | 3.96 | 0.24 | 0.41 | 0.53 |
| 0.3 | 150 | 0.13 | 0.66 | 0.98 | 1.39 | 1.68 | 1.95 | 2.30 | 2.51 | 2.82 | 3.20 | 3.90 | 4.71 | 5.88 | 2.37 | 1.20 | 0.79 | 3.99 | 0.38 | 0.60 | 0.72 |
| 0.3 | 200 | 0.36 | 1.08 | 1.34 | 1.78 | 2.09 | 2.34 | 2.60 | 2.87 | 3.22 | 3.63 | 4.25 | 4.88 | 6.27 | 2.74 | 1.22 | 0.94 | 5.96 | 0.51 | 0.74 | 0.83 |
| 0.3 | 250 | 0.87 | 1.46 | 1.76 | 2.11 | 2.38 | 2.68 | 2.97 | 3.22 | 3.52 | 3.88 | 4.48 | 4.94 | 6.33 | 3.05 | 1.12 | 0.69 | 4.33 | 0.64 | 0.85 | 0.93 |
| 0.3 | 500 | 1.89 | 2.61 | 2.94 | 3.47 | 3.77 | 4.03 | 4.33 | 4.59 | 4.85 | 5.18 | 5.74 | 6.14 | 7.38 | 4.34 | 1.08 | 0.24 | 3.27 | 0.95 | 0.99 | 1.00 |
| 0.3 | 750 | 3.13 | 3.70 | 3.99 | 4.43 | 4.80 | 5.03 | 5.34 | 5.61 | 5.89 | 6.23 | 6.77 | 7.18 | 8.28 | 5.37 | 1.07 | 0.31 | 3.24 | 1.00 | 1.00 | 1.00 |
| 0.3 | 1000 | 3.76 | 4.47 | 4.83 | 5.34 | 5.66 | 5.93 | 6.20 | 6.46 | 6.75 | 7.07 | 7.50 | 7.95 | 8.76 | 6.21 | 1.04 | 0.09 | 3.08 | 1.00 | 1.00 | 1.00 |
| 0.3 | 1500 | 5.05 | 5.85 | 6.24 | 6.69 | 7.02 | 7.30 | 7.53 | 7.77 | 8.07 | 8.47 | 8.95 | 9.31 | 10.11 | 7.56 | 1.05 | 0.09 | 3.06 | 1.00 | 1.00 | 1.00 |
| 0.3 | 2000 | 6.34 | 7.13 | 7.47 | 7.98 | 8.24 | 8.50 | 8.75 | 9.00 | 9.24 | 9.63 | 10.10 | 10.50 | 11.17 | 8.77 | 1.02 | 0.03 | 3.10 | 1.00 | 1.00 | 1.00 |
| | | | | | | | | | | | | | | | | | | | | | |
| 0.4 | 100 | -0.62 | 0.07 | 0.41 | 0.90 | 1.18 | 1.43 | 1.72 | 2.00 | 2.38 | 2.79 | 3.47 | 4.43 | 5.86 | 1.88 | 1.28 | 0.78 | 4.06 | 0.24 | 0.41 | 0.53 |
| 0.4 | 150 | 0.08 | 0.63 | 0.94 | 1.36 | 1.64 | 1.89 | 2.21 | 2.45 | 2.78 | 3.19 | 3.87 | 4.64 | 6.11 | 2.33 | 1.22 | 0.87 | 4.41 | 0.36 | 0.58 | 0.70 |
| 0.4 | 200 | 0.31 | 0.93 | 1.29 | 1.69 | 2.02 | 2.29 | 2.55 | 2.83 | 3.15 | 3.56 | 4.30 | 4.82 | 6.37 | 2.69 | 1.25 | 1.05 | 6.17 | 0.48 | 0.72 | 0.81 |
| 0.4 | 250 | 0.75 | 1.36 | 1.65 | 2.05 | 2.33 | 2.59 | 2.90 | 3.18 | 3.49 | 3.81 | 4.41 | 5.04 | 6.46 | 2.99 | 1.18 | 1.25 | 8.83 | 0.61 | 0.83 | 0.90 |
| 0.4 | 500 | 1.91 | 2.46 | 2.82 | 3.36 | 3.69 | 3.94 | 4.21 | 4.48 | 4.75 | 5.09 | 5.69 | 6.12 | 7.23 | 4.25 | 1.10 | 0.30 | 3.36 | 0.94 | 0.99 | 0.99 |
| 0.4 | 750 | 2.98 | 3.54 | 3.85 | 4.30 | 4.64 | 4.91 | 5.22 | 5.51 | 5.78 | 6.11 | 6.69 | 7.07 | 8.24 | 5.25 | 1.10 | 0.35 | 3.27 | 1.00 | 1.00 | 1.00 |
| 0.4 | 1000 | 3.63 | 4.36 | 4.71 | 5.19 | 5.53 | 5.78 | 6.03 | 6.29 | 6.62 | 6.95 | 7.39 | 7.86 | 8.72 | 6.06 | 1.06 | 0.12 | 3.13 | 1.00 | 1.00 | 1.00 |
| 0.4 | 1500 | 4.81 | 5.60 | 6.03 | 6.52 | 6.85 | 7.08 | 7.38 | 7.61 | 7.90 | 8.31 | 8.76 | 9.18 | 10.04 | 7.39 | 1.07 | 0.09 | 3.11 | 1.00 | 1.00 | 1.00 |
| 0.4 | 2000 | 6.05 | 6.92 | 7.30 | 7.75 | 8.02 | 8.29 | 8.54 | 8.80 | 9.05 | 9.44 | 9.94 | 10.33 | 11.04 | 8.57 | 1.03 | 0.03 | 3.15 | 1.00 | 1.00 | 1.00 |
| | | | | | | | | | | | | | | | | | | | | | |
| 0.5 | 100 | -0.98 | 0.04 | 0.40 | 0.81 | 1.14 | 1.39 | 1.64 | 1.94 | 2.29 | 2.72 | 3.44 | 4.34 | 5.54 | 1.82 | 1.28 | 0.62 | 3.83 | 0.23 | 0.39 | 0.50 |
| 0.5 | 150 | 0.00 | 0.60 | 0.87 | 1.25 | 1.57 | 1.82 | 2.14 | 2.44 | 2.74 | 3.17 | 3.92 | 4.77 | 6.84 | 2.31 | 1.32 | 1.17 | 5.76 | 0.35 | 0.56 | 0.67 |
| 0.5 | 200 | 0.13 | 0.84 | 1.19 | 1.58 | 1.96 | 2.20 | 2.50 | 2.76 | 3.10 | 3.55 | 4.25 | 5.01 | 6.96 | 2.63 | 1.28 | 1.06 | 6.18 | 0.48 | 0.70 | 0.78 |
| 0.5 | 250 | 0.64 | 1.24 | 1.58 | 1.96 | 2.24 | 2.51 | 2.80 | 3.08 | 3.42 | 3.84 | 4.43 | 4.99 | 6.51 | 2.92 | 1.18 | 0.77 | 4.37 | 0.57 | 0.80 | 0.89 |
| 0.5 | 500 | 1.81 | 2.34 | 2.69 | 3.19 | 3.53 | 3.84 | 4.07 | 4.32 | 4.67 | 5.03 | 5.62 | 6.03 | 7.12 | 4.13 | 1.13 | 0.36 | 3.35 | 0.91 | 0.98 | 0.99 |
| 0.5 | 750 | 2.79 | 3.34 | 3.67 | 4.12 | 4.47 | 4.74 | 5.07 | 5.32 | 5.65 | 6.00 | 6.58 | 7.02 | 8.05 | 5.10 | 1.12 | 0.37 | 3.43 | 0.99 | 1.00 | 1.00 |
| 0.5 | 1000 | 3.36 | 4.14 | 4.52 | 5.01 | 5.33 | 5.58 | 5.84 | 6.09 | 6.45 | 6.77 | 7.31 | 7.71 | 8.65 | 5.88 | 1.08 | 0.15 | 3.16 | 1.00 | 1.00 | 1.00 |
| 0.5 | 1500 | 4.66 | 5.35 | 5.76 | 6.29 | 6.59 | 6.86 | 7.12 | 7.38 | 7.69 | 8.09 | 8.59 | 9.03 | 9.67 | 7.16 | 1.09 | 0.12 | 3.02 | 1.00 | 1.00 | 1.00 |
| 0.5 | 2000 | 5.95 | 6.63 | 7.01 | 7.49 | 7.77 | 8.02 | 8.28 | 8.54 | 8.79 | 9.22 | 9.68 | 10.10 | 10.75 | 8.31 | 1.04 | 0.05 | 3.17 | 1.00 | 1.00 | 1.00 |
| 0.5 | 2500 | 6.65 | 7.43 | 7.92 | 8.34 | 8.74 | 8.99 | 9.32 | 9.58 | 9.91 | 10.25 | 10.72 | 11.11 | 11.89 | 9.31 | 1.11 | 0.05 | 2.95 | 1.00 | 1.00 | 1.00 |
| 0.5 | 5000 | 10.54 | 11.38 | 11.74 | 12.13 | 12.50 | 12.79 | 13.09 | 13.35 | 13.68 | 14.09 | 14.61 | 14.99 | 15.66 | 13.11 | 1.12 | 0.09 | 2.94 | 1.00 | 1.00 | 1.00 |
| 0.5 | 7500 | 13.70 | 14.35 | 14.75 | 15.17 | 15.49 | 15.76 | 16.02 | 16.29 | 16.61 | 16.97 | 17.42 | 17.88 | 18.72 | 16.07 | 1.07 | 0.21 | 3.17 | 1.00 | 1.00 | 1.00 |
| 0.5 | 10000 | 15.98 | 16.78 | 17.17 | 17.64 | 17.91 | 18.21 | 18.46 | 18.78 | 19.10 | 19.52 | 20.06 | 20.46 | 21.18 | 18.55 | 1.12 | 0.15 | 2.88 | 1.00 | 1.00 | 1.00 |