
FORECASTING TIME SERIES WITH BOOT.EXPOS PROCEDURE *

Authors: CLARA CORDEIRO
– Dept. Math, FCT, University of Algarve, Portugal
ccordei@ualg.pt
M. MANUELA NEVES
– CEAUL and Dept. Math, ISA, TULisbon, Portugal
manela@isa.utl.pt

Abstract:

- To forecast future values of a time series is one of the main goals in times series analysis. Many forecasting methods have been developed and its performance evaluated. In order to make a selection among an avalanche of such emerging methods they have to be compared in a kind of forecasting competition. One of these competitions is the M3 competition with its 3003 time series. The competition results in Makridakis and Hibon (2000) paper are frequently used as a benchmark in comparative studies. The Boot.EXPOS approach developed by the authors, combines the use of exponential smoothing methods with the bootstrap methodology to forecast time series. The idea is to join these two approaches (bootstrap and exponential smoothing) and to construct a computational algorithm to obtain forecasts. It works in an automatic way and can be summarized as follows: (i) choose an exponential smoothing model, among several proposed using the mean squared error, and obtain the model components; (ii) fit an AR to the residuals of the adjusted model; the order of the AR is selected by AIC criterion; (iii) center the new residuals obtained in previous step and resample; (iv) obtain a bootstrapped replica of the time series according to the AR model and exponential smoothing components found in first step; (v) forecast future values according to model in (i); (vi) compute the point forecast as the mean or as the median of the predicted values. The performance of the procedure here proposed is evaluated by comparing it with other procedures presented in the M3 competition. Some accuracy measures are used for that comparison. All computational work is done using the R2.8.1 software (R Development Core Team, 2008).

Key-Words:

- *bootstrap; exponential smoothing; forecasting accuracy; M3 competition.*

AMS Subject Classification:

- 62F40, 60G25, 37M10.

*Paper accepted for presentation at COMPSTAT 2008.

1. INTRODUCTION

In our days it is well known the importance of time series studies. These studies provide indicators about a country economy, the unemployment rate, the export and import product rates, etc. A time series is a set of observations usually ordered in equally spaced intervals. The first step in the analysis of any time series is the description of the historic series. It includes the graphical representation of the data. When a time series is plotted, common patterns are frequently found. These patterns might be explained by many possible cause-and-effect relationships. Common components are the trend, seasonal effect, cyclic changes and randomness. The more interesting and ambitious task is to forecast future values of a series on the basis of its recorded past, and more specifically to calculate forecast intervals. Classical procedures to obtain forecast intervals assume that the distribution of the error process is known. Bootstrap approaches have been proposed to compute distribution free forecast intervals. The authors here propose a procedure (*Boot.EXPOS*) to forecast time series that is inspired on sieve bootstrap approach (Bühlmann, 1997). The *Boot.EXPOS* procedure starts by selecting the best exponential smoothing method according to the characteristics that a times series reveals, among a set of methods. After adjusting the best model, our attention is drawn to the residual part. The bootstrap is then used after an autoregressive adjustment, selected by AIC criterion. The time series is then reconstructed, adding the initial components (if they exist) to the bootstrapped residuals, see Cordeiro and Neves (2007a, 2007b) for more details. Forecasts are finally obtained using the model initially selected. The procedure runs automatically, so its possible to test it on large data sets. Surely there are advantages and disadvantages in this automatic process, but for now we are not discussing this. The issue here is to discuss measures of its performance, results and progress achieved. The computational work was done using the R software. Some of the R packages and functions were used, but new functions needed to be constructed.

2. EXPONENTIAL SMOOTHING METHODS

Exponential smoothing (*EXPOS*) refers to a set of forecasting methods, several of which are widely used. The *EXPOS* is a procedure that continually updates a forecast emphasizing the most recent experience, that is, recent observations are given more weight than the older observations. Single exponential smoothing, Holt's linear trend, Holt-Winters seasonal smoothing with either additive or multiplicative seasonality are some examples of *EXPOS* methods, see DeLurgio (1998) for more details. The forecasting performance of exponential

smoothing methods has been addressed by several authors. A very good reviewing of the past 25 years of time series forecasting is given by De Gooijer and Hyndman (2006). These methods are relatively simple but reveal robust approaches to forecasting and are accurate in model identification. The classical Box–Jenkins ARIMA models require the user to identify an appropriate model and to use at least 50 observations to have a good chance of success (Chatfield, 1978).

Table 1 shows four EXPOS methods addressed here, where data with or without trend and/or with or without seasonal components are considered. The goal is to choose the EXPOS model using the mean squared error criterion and then separate the pattern (trend or/and seasonality) components from the error term.

Table 1: The EXPOS methods considered.

Classification	Method
1	Single exponential smoothing
2	Holt’s linear trend
3	Holt–Winters seasonal smoothing with additive seasonality
4	Holt–Winters seasonal smoothing with multiplicative seasonality

2.1. Holt–Winters method

The Holt–Winters forecasting method is applied whenever the data behavior is trendy and is seasonally. Relatively to the seasonal factor it can be additive or multiplicative, depending on the oscillatory movement along the time period. The *additive* Holt–Winters (classification 3 in Table 1) has the following recursive equations to estimate the trend and the seasonal factor at time t

$$T_t = \alpha(X_t - S_{t-s}) + (1 - \alpha)(T_{t-1} + b_{t-1})$$

$$b_t = \beta(T_t - T_{t-1}) + (1 - \beta)b_{t-1}$$

$$S_t = \gamma(X_t - T_t) + (1 - \gamma)S_{t-s}$$

with $\alpha, \beta, \gamma \in [0, 1]$ and

T_t smoothed value at end of period t after adjusting for seasonality

X_t value of actual demand at end of period t

S_{t-s} smoothed seasonal index, s periods ago

b_t smoothed value of trend through period t

- α smoothing constant used for T_t
- β smoothing constant used to calculate the trend (b_t)
- γ smoothing constant used for calculate the seasonal index in period t .

The prediction equation is

$$(2.1) \quad \hat{X}_t(h) = T_t + h \times b_t + S_{t+h-rs}$$

where $h = 1, 2, 3, \dots$ is the forecast horizon and $r = 1$ if $1 \leq h \leq s$, $r = 2$ if $s < h \leq 2s$, etc.

The *multiplicative* Holt–Winters (classification 4 in Table 1) has as recursive equations

$$\begin{aligned} T_t &= \alpha(X_t/S_{t-s}) + (1 - \alpha)(T_{t-1} + b_{t-1}) \\ b_t &= \beta(T_t - T_{t-1}) + (1 - \beta)b_{t-1} \\ S_t &= \gamma(X_t/T_t) + (1 - \gamma)S_{t-s} \end{aligned}$$

and prediction equation

$$(2.2) \quad \hat{X}_t(h) = (T_t + h \times b_t) \times S_{t+h-rs} ,$$

where the parameters are defined above.

The exponential smoothing parameters (α, β, γ) are estimated by minimizing the sum of squared errors and are restricted to values in (0,1). Simple exponential smoothing and Holt's method (classification 1 and 2 in Table 1) are derived from the above equations considering the corresponding exponential smoothing parameters (β, γ) to be set to zero.

3. ABOUT BOOTSTRAP

The bootstrap resampling technique (Efron, 1979) is a very popular methodology in independent data because of its simplicity and nice properties. It is a computer-intensive method that presents solutions in situations where the traditional methods fail. Efron's bootstrap classical approach has revealed inefficient in the context of dependent data, such as in the case of time series, where the dependence data arrangement should be kept during the resampling scheme. But for dependent data the generating process is often not fully specified. Then there exists no unique way for resampling.

A great development in the resampling methods area for dependent data has been observed, see Lahiri (2003). The majority of those methods suggests the use of blocks, in order to keep the dependence structure. Different versions

of blocking differ in the way as blocks are constructed. The most well known versions are: Nonoverlapping Block Bootstrap (Carlstein, 1992); Moving Block Bootstrap (Künsch, 1989); Circular Block Bootstrap (Politis and Romano, 1992) and Stationary Block Bootstrap (Politis and Romano, 1994). For a large class of stationary processes, Bühlmann (1997) presents the sieve bootstrap method based on a sieve of autoregressive processes of increasing order. Recently Alonso *et al.* (2002) extended the sieve approach in order to obtain prediction intervals. In Cordeiro and Neves (2006) several bootstrap methodologies for dependent data were compared in constructing forecast intervals and the sieve bootstrap has revealed as a good compromise for obtaining forecast intervals.

3.1. Particular approach: sieve bootstrap

In 1997, Bühlmann proposed a bootstrap scheme called sieve bootstrap. This method is based on the idea of fitting parametric models first and afterwards resampling from the residuals. However the model is chosen adaptively rather than considering a pre-fixed model. This approach is different from other bootstrap methods, the sample bootstrap is (conditionally) stationary and does not present structure of dependence. Another different feature is that the sieve bootstrap sample is not a subsample from the original data, as in other methods. Given a sample X_1, \dots, X_n , from a stationary process, select the order $p = p(n)$ of an autoregressive approximation by AIC criterion. The autoregressive coefficients are Yule–Walker estimates. The AR(p) model is used to filter the residuals series. The residuals are then centered and the empirical cumulative distribution function of these residuals is obtained. From this distribution we get an i.i.d. resample of the centered residuals. Use the AR for obtaining a new series X_t^* by recursion. Given X_1^*, \dots, X_T^* compute the estimation of the autoregressive coefficients and then obtain future bootstrap observations by recursion from the new series.

3.2. Boot.EXPOS procedure

Here a different approach is proposed: first fit an EXPOS model to the data and then to proceed like the above procedure over the residuals — Boot.EXPOS (Cordeiro and Neves, 2008). The Boot.EXPOS procedure starts by selecting the best forecasting method, section 2. The seasonality and trend components (if they exist) are removed from the initial series and only added at the end to reconstruct the series. In between, the procedures go on like the sieve bootstrap approach. This general steps are described in Table 2, where it is also established a comparison between the previous and the new approach.

Table 2: The previous and the new approach.

Sieve bootstrap	Boot.EXPOS
<p>Step 1: Adjust an autoregressive model with increasing order p using AIC criterion;</p> <p>Step 2: Obtain the residuals;</p> <p style="text-align: center;"><i>For B replicates:</i></p> <p>Step 3: Resample the centered residuals;</p> <p>Step 4: Use AR for obtaining a new series by recursion;</p> <p>Step 5: Fit $AR(p)$ to the new series;</p> <p>Step 6: Obtain the predicted values from the new series using the previous $AR(p)$ fit.</p>	<p>Step 0: Select the best EXPOS method; components are removed and the residuals obtained;</p> <p>Step 1: Adjust an autoregressive model with increasing order p using AIC criterion;</p> <p>Step 2: Obtain the residuals;</p> <p style="text-align: center;"><i>For B replicates:</i></p> <p>Step 3: Resample the centered residuals;</p> <p>Step 4: Use AR for obtaining a new series by recursion;</p> <p>Step 5: Add the components in Step 0 to the new series; fit EXPOS method (same type as in Step 0);</p> <p>Step 6: Obtain the predicted values from the new series using the previous EXPOS fit.</p>

3.2.1. Dealing with some statistical issues

The initial step in the Boot.EXPOS procedure is to fit an EXPOS method selected by MSE, section 2. Then the random part is separated from the other patterns, such as trend or/and seasonality (these patterns are added at the end). Next, a test on the stationarity of the random part is performed, before the AR adjustment. If the stationarity is not accepted, transformation to the data is required.

Statistical tests used to study stationarity and data transformations:

- Test for a unit root in a time series when the model under consideration in the null hypothesis does not present autocorrelation in the error term. In such case, simple version of the Dickey–Fuller test is the most appropriate (Halkos and Kevork, 2005).
- Box–Cox transformations (Box and Cox, 1964) can impose stability on data variance stability and can make seasonal effect additive. Transformation is used whenever data exhibit multiplicative seasonality, *i.e.*,

when EXPOS model choice is 4 according to Table 1. Although the use of these transformations are not worthwhile in some cases, for example in economic data (Nelson and Granger, 1979), there can be some advantages in using it. Computational work with $\lambda = 0$, $-1 \leq \lambda \leq 1$, $0 \leq \lambda \leq 1$, $0 \leq \lambda \leq 0.5$, $0 \leq \lambda \leq 0.9$ and $0 \leq \lambda \leq 0.99$, was performed and it has revealed a good option when $0 \leq \lambda \leq 1$.

- Differencing can transform a non-stationary series to a stationary series. The KPSS (Kwiatkowski *et al.*, 1992) procedure tests for the null hypothesis that a time series has a stationary root against a unit-root.

Boot.EXPOS application makes all these procedures whenever the non-stationarity is detected. But if the data goes over to the above processes and the presence of the non-stationarity is still detected in the data, the best EXPOS method is used to obtain forecast without bootstrapping.

4. FORECAST ACCURACY MEASURES

To evaluate the performance of Boot.EXPOS procedure some accuracy measures are used. In M3 competition some of those measures are calculated. Concerning this subject it is also interesting to read the article proposed by Hyndman and Koehler (2006).

Let X_t denote the observation at time t and \hat{X}_t the forecast of X_t . The forecast error is defined by $e_t = X_t - \hat{X}_t$. The forecasts are computed for a hold-out period. Thus the out-of-sample forecasts $\hat{X}_n(1), \dots, \hat{X}_n(h)$ are computed based on the data from time $t = 1, \dots, n$. Accuracy measures are then computed in order to compare our results with those presented in Makridakis and Hibon (2000) in <http://www.forecastingprinciples.com/m3-competition.html>. The following accuracy measures are here considered:

Table 3: Two accuracy measures.

Acronyms	Name	Definition
sMAPE	Symmetric Mean Absolute Percentage Error	$\text{mean}\left(200 \frac{ e_t }{X_t + \hat{X}_t}\right)$
RMSE	Root Mean Squared Error	$\sqrt{\text{mean}(e_t^2)}$

5. A COMPETITIVE EXAMPLE

In this section, the performance and evaluation of the *Boot.EXPOS* procedure is analyzed. A 3003 time series data — M3 Competition — is used in this analysis. This huge data set has been used by many researchers as a powerful tool to test new forecasting methods and this is what we shall present next.

5.1. The competing data

The M3 Competition data set involves 3003 time series selected on a quota basis of 6 different types of series: micro, industry, finance, demographic and other; and 4 different time intervals between successive observations: yearly, quarterly, monthly and other. The historical values of each series are at least 14 observations for yearly data, 16 for quarterly data, 48 for monthly data and 60 observations for other data. The time horizons of forecasting are 6 periods for yearly data, 8 periods for quarterly and other data, and 18 periods for monthly data. Table 4 shows the number of series in each category and Figure 1 shows the plot of examples of some types.

Table 4: The 3003 time series distribution.

Period	Type of times series data						Total
	Demographic	Finance	Industry	Macro	Micro	OTHER	
Monthly	111	145	334	312	474	52	1428
OTHER	0	29	0	0	4	141	174
Quarterly	57	76	83	336	204	0	756
Yearly	245	58	102	83	146	11	645
Total	413	308	519	731	828	204	3003

5.2. Some tools use in competition

An automatic procedure to analyze each time series is necessary as there is a large set of time series. All the intensive computational work is performed using R 2.8.1 software (R Development Core Team, 2008). Packages such as **car**, **FitAR**, **forecast**, **Mcomp**, **tseries**, among others, are widely used in the implementation of the new procedures in R language: **best.EXPOS()** to select the best EXPOS method and **boot.EXPOS()** a procedure using bootstrap and EXPOS.

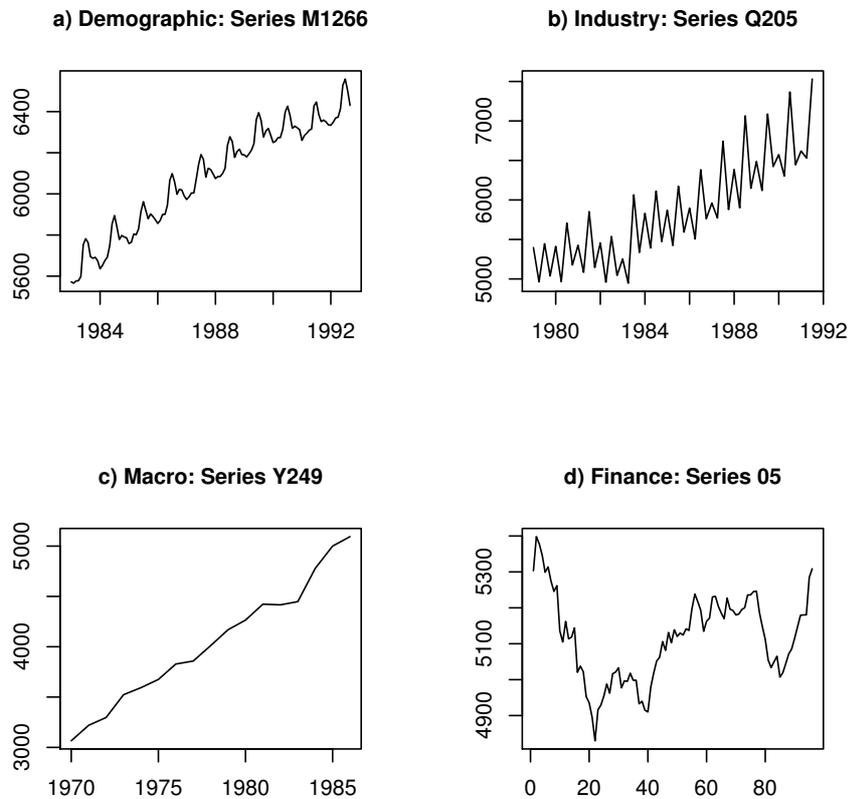


Figure 1: M3 competition category: monthly (a), quarterly (b), yearly (c) and “other” (d).

5.3. EXPOS vs Boot.EXPOS

All time series considered are strictly positive. For each category, different forecasting periods are considered: 6 for yearly, 8 for quarterly and “other”, 18 for monthly. For the M3 competition a comparison between the forecasts produced by any of the four EXPOS methods and the forecast produced by Boot.EXPOS procedure is showed in Figure 2. Symmetric mean absolute percentage error is one of the accuracy measures used in this study. It can be seen that for the time period interval yearly and Other the resampling technique is not a favorable procedure. In the monthly and quarterly cases the scenario is better. It seems that for time series with components trend and seasonality the Boot.EXPOS procedure can be a good compromise in forecasting.

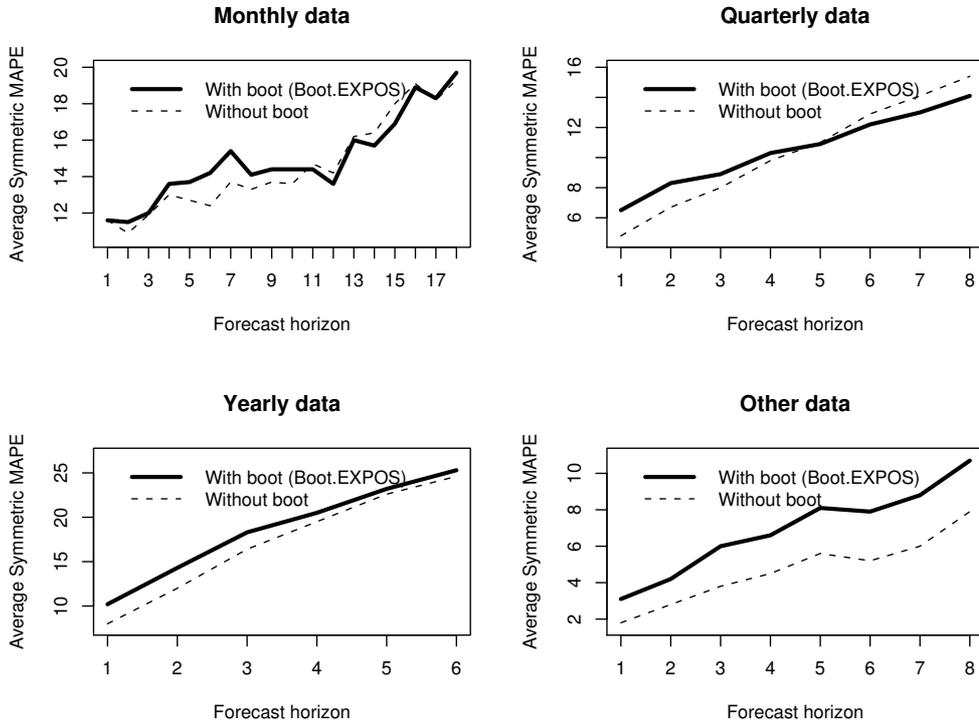


Figure 2: EXPOS and Boot.EXPOS comparison using M3 competition data.

5.4. Boot.EXPOS vs six competition methods

Each M3 competition series was classified according to Table 1, using function `best.EXPOS()`. This function selects the model that presents the minimum MSE. For monthly time series the distribution is showed in Table 5. Here, to illustrate Boot.EXPOS only **monthly time series** will be considered. For each series the procedure scheme in Table 2, right, goes from Step 0 to Step 6. One thousand replications ($B = 1000$) are carried out and forecasts are obtained for each replication. Two forecast estimates are obtained: the average and median of the B forecasted values. Only the average is used here and is based in the 95% central forecast simulations.

Table 5: Number of monthly time in each method.

Category	Classification			
	1	2	3	4
Monthly	515	300	345	268

In these competitions the participating experts are asked to obtain a given number of forecast values for each data period. Their forecasts are latter compared with the actual values and the accuracy of such forecasts is calculated. Makridakis and Hibon (2000) lists 24 methods of forecasting used in the M3 competition. Naive2, Box–Jenkins automatic, ForecastPro, THETA, RBH and ForecastX are the six methods that present the best performance in M competition (Makridakis *et al.*, 1982) and M3 competition (Makridakis and Hibon, 2000). These well behaved 6 methods (Hyndman *et al.*, 2002) are used in comparison to our approach. For the monthly case the sMAPE and RMSE are present in Tables 6 and 7, respectively.

Table 6: Average symmetric MAPE: 1428 monthly series.

Method	Forecasting horizon																	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Naive2	15.0	13.5	15.7	17.0	14.9	14.4	16.0	15.6	15.9	16.0	16.7	16.0	18.1	18.4	19.3	21.3	19.6	20.7
B-J automatic	12.3	11.7	12.8	14.3	12.7	12.3	13.6	13.0	13.4	13.4	14.5	14.1	16.2	16.9	17.8	19.7	18.1	19.3
ForecastPRO	11.5	10.7	11.8	13.0	11.9	12.0	13.0	12.7	13.0	13.0	13.9	13.3	15.3	15.4	16.4	18.2	16.8	18.3
THETA	11.2	10.7	11.8	12.4	12.2	12.2	13.6	12.7	13.2	13.4	13.5	13.2	15.4	15.3	16.5	17.9	17.0	18.5
RBF	13.7	12.3	13.7	14.3	12.3	12.5	14.6	13.5	14.2	14.5	14.1	14.1	16.1	15.8	17.3	18.3	16.8	17.8
ForecastX	11.6	11.2	12.6	14.0	12.4	12.0	13.0	12.8	13.1	13.4	13.9	14.0	15.8	16.6	17.8	19.4	18.1	18.8
Boot.EXPOS	11.6	11.5	12.0	13.6	13.7	14.2	15.4	14.1	14.4	14.4	14.4	13.6	16.0	15.7	16.9	18.9	18.3	19.7

Table 7: Root mean squared error: 1428 monthly series.

Method	Forecasting horizon											Average 1–18
	1	2	3	4	5	6	8	12	15	18		
Naive2	1144	1367	1466	1643	1363	1201	1453	1329	1766	1673	1448	
B-J automatic	864	942	934	1061	1006	1100	1107	1208	1454	1563	1185	
ForecastPRO	812	905	913	1068	1032	990	1157	1135	1411	1463	1146	
THETA	810	936	1067	1181	1130	979	1170	1138	1445	1487	1168	
RBF	984	1636	1468	1850	1503	1000	1355	1197	1764	1651	1459	
ForecastX	794	977	920	1087	1008	966	1175	1169	1457	1510	1163	
Boot.EXPOS	840	988	1214	1627	1370	1540	1702	1149	1580	1828	1405	

In a first look the results have a reasonable classification among the six methods in study. But if the results are separated into classification 1, 2, 3 and 4 (Table 1) the scenario gets better. Results for the above six methods and for the Boot.EXPOS procedure are given in Tables 8, 9, 10 and 11, corresponding to each classification.

The performance of the various methods depends upon the length of the forecasting horizon. Our method presents sMAPE among the values of the selected methods (Table 6), particularly it has a very good performance when the time series model classification is 4 (Table 11). When the time series model is the Holt–Winters additive (classification 3) Table 10 shows a reasonable behavior. Regarding the other classification (1 and 2) Tables 8 and 9 show poor results.

It is evident that the procedure is best succeeded when the time series is seasonal and trendy. For time series in classifications 1 and 2, the authors thought that the classical EXPOS methods could produce accurate forecasts but in fact a simulation computational study has revealed the opposite. For some of these time series whenever stationarity was achieved the Boot.EXPOS forecasts were better values.

Table 8: Average symmetric MAPE: 1.

Method	Forecasting horizon																	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Naive2	16.2	16.2	17.9	20.5	18.5	18.4	20.8	20.4	19.2	19.3	20.4	19.4	21.5	22.2	23.5	26.3	22.7	26.0
B-J automatic	12.3	12.9	15.3	16.8	16.3	16.3	17.4	18.0	18.3	17.0	18.4	17.1	20.4	20.5	22.6	23.4	21.5	24.7
ForecastPRO	12.1	12.4	14.5	16.1	15.3	15.9	16.6	17.8	17.2	16.5	17.8	16.4	20.0	19.8	21.7	23.0	20.7	23.8
THETA	12.0	12.8	15.0	15.9	15.8	15.8	17.7	17.9	16.8	16.5	17.3	16.1	19.9	19.6	21.3	22.6	19.9	23.5
RBF	14.9	14.8	16.8	18.3	16.2	16.8	18.7	18.7	17.2	17.3	17.6	17.0	20.4	20.0	22.3	23.5	20.4	22.8
ForecastX	11.1	12.9	14.3	16.2	15.3	15.9	16.6	17.7	16.6	16.4	17.2	16.5	19.4	19.6	21.9	23.2	20.7	23.3
Boot.EXPOS	14.8	14.8	15.5	17.6	16.5	15.9	17.2	17.2	16.8	17.0	19.2	17.4	20.1	21.2	21.7	24.4	21.8	24.4

Table 9: Average symmetric MAPE: 2.

Method	Forecasting horizon																	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Naive2	1.4	2.0	2.7	3.8	3.9	4.3	4.9	5.2	5.4	5.9	6.5	7.2	7.7	7.6	8.6	9.1	9.5	10.1
B-J automatic	1.3	1.7	2.3	3.4	3.5	3.7	4.3	4.6	4.7	5.2	5.6	6.2	6.6	6.6	7.5	8.2	8.5	9.0
ForecastPRO	1.3	1.7	2.1	3.2	3.2	3.6	4.0	4.4	4.5	5.0	5.6	5.9	6.5	6.3	7.5	8.1	8.6	9.3
THETA	1.3	1.7	2.3	3.3	3.3	3.6	4.2	4.4	4.6	5.1	5.8	6.3	6.7	6.7	7.8	8.2	8.6	9.2
RBF	3.0	3.3	3.7	4.3	3.9	4.1	4.7	4.9	4.8	5.4	6.0	6.2	6.6	6.4	7.3	7.6	7.7	8.3
ForecastX	1.3	1.7	2.3	3.4	3.5	3.8	4.5	4.8	5.0	5.5	6.0	6.6	7.1	7.2	8.2	8.8	9.3	9.9
Boot.EXPOS	4.6	5.0	5.8	6.9	6.2	7.7	7.6	7.8	8.3	7.6	8.3	9.2	10.6	10.5	12.0	11.7	11.6	12.3

Table 10: Average symmetric MAPE: 3.

Method	Forecasting horizon																	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Naive2	12.8	13.3	13.9	16.0	13.7	12.7	14.8	13.8	15.4	15.7	16.7	14.0	16.8	15.7	17.6	17.3	17.4	18.3
B-J automatic	11.0	11.4	10.5	12.6	11.4	10.7	12.3	11.6	13.2	12.7	13.8	12.7	14.3	14.1	15.7	15.8	15.2	16.2
ForecastPRO	10.5	10.6	10.4	12.4	11.0	10.7	11.9	11.3	13.3	12.9	13.7	12.0	13.6	13.4	14.8	15.6	14.8	15.9
THETA	10.5	10.6	10.7	12.2	11.6	11.0	12.5	11.4	14.1	12.7	14.1	12.2	13.6	13.3	15.0	15.2	15.3	16.7
RBF	12.8	12.5	12.3	13.5	11.1	11.2	14.3	12.3	14.7	14.8	15.3	13.6	16.1	15.1	16.6	17.0	16.1	17.1
ForecastX	10.7	10.9	10.4	12.7	11.4	10.4	12.3	11.6	13.2	12.9	14.3	12.9	14.5	14.0	15.9	15.6	16.3	16.7
Boot.EXPOS	9.5	10.3	10.9	12.8	14.6	15.4	17.9	15.7	16.4	13.6	13.4	12.0	13.9	12.2	15.5	16.8	17.7	19.2

Table 11: Average symmetric MAPE: 4.

Method	Forecasting horizon																	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Naive2	23.7	18.1	23.0	22.8	19.3	18.5	19.6	19.3	19.7	19.4	19.8	20.1	22.5	24.0	23.7	27.7	25.0	24.8
B-J automatic	19.8	16.9	18.9	20.0	16.2	15.6	17.3	15.3	14.8	15.9	17.3	17.4	20.3	22.6	21.6	26.8	23.6	23.5
ForecastPRO	17.9	14.6	16.5	16.7	14.9	14.8	16.2	14.7	14.4	15.0	15.8	16.3	18.1	19.0	18.8	22.5	20.2	21.2
THETA	16.8	14.4	15.8	15.1	15.0	15.2	16.7	14.4	14.6	16.5	14.5	15.8	18.5	18.6	19.0	21.9	21.1	21.6
RBF	19.6	15.4	18.3	17.6	15.2	15.2	17.4	15.4	16.7	17.2	15.2	16.6	18.3	18.6	19.7	21.5	19.7	19.8
ForecastX	18.6	15.4	19.1	19.4	16.1	15.0	15.8	14.6	14.8	16.2	15.6	17.2	19.0	22.0	21.9	25.6	22.8	22.2
Boot.EXPOS	16.2	14.0	13.6	14.2	15.7	16.5	17.2	13.4	14.2	18.1	13.5	13.7	17.0	15.5	15.2	18.9	20.0	19.7

6. CLOSING COMMENTS

An automatic procedure based on EXPOS and bootstrap methodology is presented. Our methodology was applied to the M3 competition data. Accuracy measures such as sMAPE, RMSE were calculated and compared with the measures obtained for the best six competition methods. The method revealed a very good performance for series with seasonal and trendy components. Short length time series showed difficulties in the forecasting procedure. In fact Billah *et al.* (2006) decide to separate the short series in order to obtain... *plausible results with satisfactory level of statistical reliability.*

Some research is in progress for improving the results obtained, as well as for considering new methods for the initial fitting.

ACKNOWLEDGMENTS

The authors are grateful to Professor Michèle Hibon for kindly providing the forecasts submitted to the M3 competition.

REFERENCES

- [1] ALONSO, A.M.; PEÑA, D. and ROMO, J. (2002). Forecasting time series with sieve bootstrap, *Journal of Statistical Planning and Inference*, **100**, 1–11.
- [2] BILLAH, B.; KING, M.; SNYDER, R. and KOEHLER, A. (2006). Exponential smoothing model selection for forecasting, *International Journal of Forecasting*, **22**, 239–247.
- [3] BOX, E.P. and COX, D.R. (1964). An analysis of transformations, *Journal of the Royal Statistical Society*, **B**.
- [4] BÜHLMANN, P. (1997). Sieve bootstrap for time series, *Bernoulli*, **3**, 123–148.
- [5] CARLSTEIN, E. (1992). Resampling techniques for stationary time-series: some recent developments, *New Directions in Time Series Analysis*, **27**, 75–85.
- [6] CHATFIELD, C. (1978). The Holt–Winters forecasting procedure, *Applied Statistics*, **27**(3), 264–279.
- [7] CORDEIRO, C. and NEVES, M.M. (2006). *The Bootstrap methodology in time series forecasting*. In “Proceedings of CompStat2006” (J. Black and A. White, Eds.), Springer Verlag, 1067–1073.

- [8] CORDEIRO, C. and NEVES, M.M. (2007a). *The Bootstrap prediction intervals: a case-study*. In “Proceedings of the 22nd International Workshop on Statistical Modelling (IWSM2007)” (J. Castillo, A. Espinal and P. Puig, Eds.), Springer Verlag, 191–194.
- [9] CORDEIRO, C. and NEVES, M.M. (2007b). *Resampling techniques in time series prediction: a look at accuracy measures*. In “Proceedings of the 56th Session of the International Statistical Institute (ISI 2007)” (M.I. Gomes, D. Pestana and P. Silva, Eds.), Springer Verlag, 353 in CD-ROM.
- [10] CORDEIRO, C. and NEVES, M.M. (2008). *Bootstrap and exponential smoothing working together in forecasting time series*. In “Proceedings in Computational Statistics (COMPSTAT 2008)” (Paula Brito, Editor), Physica-Verlag, 891–899 in CD-ROM.
- [11] DE GOOIJER, J.G. and HYNDMAN, R.J. (2006). 25 years of time series forecasting, *International Journal of Forecasting*, **22**, 443–473.
- [12] DELURGIO, S.A. (1998). *Forecasting Principles and Applications*, McGraw-Hill International Editions.
- [13] EFRON, B. (1979). Bootstrap methods: another look at the Jackknife, *The Annals of Statistics*, **7**, 1–26.
- [14] HALKOS, G. and KEVORK, I. (2005). A comparison of alternative unit root tests, *Journal of Applied Statistics*, **32**(1), 45–60.
- [15] HYNDMAN, R.; KOEHLER, A.; SNYDER, R. and GROSE, S. (2002). A state framework for automatic forecasting using exponential smoothing methods, *International Journal of Forecasting*, **18**, 439–454.
- [16] HYNDMAN, R. and KOEHLER, A. (2006). Another look at measures of forecast accuracy, *International Journal of Forecasting*, **22**, 679–688.
- [17] KÜNSCH, H. (1989). The Jackknife and the Bootstrap for general stationary observations, *The Annals of Statistics*, **17**(3), 1217–1241.
- [18] KWIATKOWSKI, D.; PHILLIPS, P.; SCHMIDT, P. and SCHIN, Y. (1992). Testing the null hypothesis of stationarity against the alternative of a unit root, *Journal of Econometrics*, **54**, 159–178.
- [19] LAHIRI, S.N. (2003). *Resampling Methods for Dependent Data*, Springer Verlag.
- [20] MAKRIDAKIS, S.; ANDERSON, A.; CARBONE, R.; FILDES, R.; HIBON, M.; LEWANDOWSKI, R.; NEWTON, J.; PARZEN, E. and WINKLER, R. (1982). The accuracy of extrapolation (time series) methods: results of a forecasting competition, *Journal of Forecasting*, **1**, 111–153.
- [21] MAKRIDAKIS, S. and HIBON, M. (2000). The M3-Competition: results, conclusions and implications, *International Journal of Forecasting*, **16**, 451–476.
- [22] NELSON, H.L. and GRANGER, C. (1979). Experience with using the Box–Cox transformation when forecasting economic time series, *Journal of Econometrics*, **10**, 57–69.
- [23] POLITIS, D.R. and ROMANO, J.P. (1992). *A Circular Block-Resampling Procedure for Stationary Data*, Wiley, New York, Springer, 225–270.
- [24] POLITIS, D.R. and ROMANO, J.P. (1994). The stationary bootstrap, *Journal of the American Statistical Association*, **89**(428), 1303–1313.
- [25] R DEVELOPMENT CORE TEAM (2008). *R: A language and environment for statistical computing*, R Foundation for statistical computing, Vienna, Austria, <http://CRAN.R-project.org>.