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BAYESIAN ESTIMATION OF THE EXPONENTIATED GAMMA PARAMETER AND RELIABILITY FUNCTION UNDER ASYMMETRIC LOSS FUNCTION

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Abstract:

- In this paper, we propose Bayes estimators of the parameter of the exponentiated gamma distribution and associated reliability function under General Entropy loss function for a censored sample. The proposed estimators have been compared with the corresponding Bayes estimators obtained under squared error loss function and maximum likelihood estimators through their simulated risks (average loss over sample space).

Key-Words:

- *exponentiated gamma distribution; General Entropy loss function; censored samples; Bayes estimators; simulated risks.*

AMS Subject Classification:

- 62F15, 62F10.

1. INTRODUCTION

The exponential distribution has been extensively used in life data analysis, but it is suitable for those situations where hazard rate is constant. For monotonic hazard rate, a number of distributions have been proposed and perhaps the most widely used among these are Weibull and gamma distributions. Both of these distributions have increasing/decreasing hazard rate depending on their shape parameters. However, one major disadvantage of the gamma distribution is that its distribution function and survival function can not be expressed in nice closed forms, particularly, if the shape parameter is not an integer. Even if the shape parameter is an integer, the hazard function involves the incomplete gamma function which is difficult for further mathematical manipulations. Numerical integration is often used to obtain the distribution function, the survival function or the hazard function. This may be one of the reasons that made the gamma distribution unpopular in comparison to the Weibull distribution. Although Weibull distribution has a nice closed form for hazard and survival function, it has its own disadvantages. For example, Bain and Engelhardt [1] have pointed out that the maximum likelihood estimators (MLE's) for the parameters of the Weibull distribution may not behave properly over the whole parameter space. Gupta *et al.* [5] proposed the use of the exponentiated gamma distribution as an alternative to gamma and Weibull distributions. The probability density function (p.d.f.) of the exponentiated gamma (EG) distribution is given below

$$f(t|\theta) = \theta\, t\, e^{-t}\big[1-e^{-t}(t+1)\big]^{\theta-1}, \qquad t>0, \quad \theta>0 , \tag{1.1}$$

where θ is the shape parameter of the distribution. The cumulative distribution function (*c.d.f.*) and the reliability function, denoted as $F(x)$ and $R(x)$, of the distribution having p.d.f. (1.1) are given as

$$F(x) = \big[1-e^{-x}(x+1)\big]^{\theta} \tag{1.2}$$

and

$$R(x) = 1-\big[1-e^{-x}(x+1)\big]^{\theta} . \tag{1.3}$$

It may be noted here that the considered model is a simple generalization of the Gamma distribution with known shape and scale parameters, namely G(2,1). This distribution is parsimonious in parameters and, hence, simple to use. The other advantage is that it has various shapes of hazard function for different values of θ. It has increasing hazard function when $\theta > 1/2$ and its hazard function takes Bath-tub shape for $\theta \le 1/2$. For other details about this distribution, we refer Shawky and Bakoban [9].

For the estimation of the parameter of a distribution, it is most common to use quadratic loss, defined as

$$L_1(\theta,\hat{\theta}) = (\hat{\theta}-\theta)^2 , \tag{1.4}$$

where $\hat{\theta}$ is the estimate of θ. It may be noted here that (1.4) defines a symmetric loss function which may be suitable for estimation of location parameter. For scale parameter, a modified form of this may be defined as follows

$$L_2(\theta,\hat{\theta}) = \left(\frac{\hat{\theta}}{\theta}-1\right)^2. \tag{1.5}$$

One can criticize the use of the quadratic loss function L_2 for the scale parameter estimation, because it penalizes overestimation more heavily. An alternative loss function may be defined on the basis of the Kullback–Leibler information number. Kullback [7] described the entropy distance as the mean information from the likelihood function $f(\mathbf{t},\theta)$ against $f(\mathbf{t},\hat{\theta})$, where $\mathbf{t}=(t_1,t_2,...,t_n)$, and, thus, the loss function may be defined as

$$L_3(\theta,\hat{\theta}) = E\left[\ln\frac{f(\mathbf{t},\hat{\theta})}{f(\mathbf{t},\theta)}\right]. \tag{1.6}$$

Accordingly, it reduces for the distribution (1.1) as

$$L_3(\hat{\theta},\theta) \propto \left(\frac{\hat{\theta}}{\theta}\right) - \ln\left(\frac{\hat{\theta}}{\theta}\right) - 1\ . \tag{1.7}$$

This loss function is known as Entropy loss function and it was first introduced by James and Stein [6] for the estimation of the Variance-Covariance (i.e., Dispersion) matrix of the Multivariate normal distribution. Dey *et al.* [4] considered this loss function for simultaneous estimation of scale parameters and their reciprocals, for p independent gamma distributions. Rukhin and Ananda [8] considered the estimation problem of the variance of a Multivariate Normal vector under the Entropy loss and Quadratic loss. The loss function (1.6) has also been used by many other authors (see Yang [11], Wieczorkowski and Zielinski [10], etc.). Calabria and Pulcini [2] defined General Entropy loss function (GELF) as

$$L(\theta,\hat{\theta}) \propto \left(\frac{\hat{\theta}}{\theta}\right)^{c_1} - c_1\ln\left(\frac{\hat{\theta}}{\theta}\right) - 1\ . \tag{1.8}$$

The constant c_1 involved in (1.8) is its shape parameter. It reflects the departure from symmetry. When $c_1>0$, over estimation ($\hat{\theta}>\theta$) is considered to be more serious than under estimation of equal magnitude and vice versa. Needless to mention that the loss (1.8) is a generalization of the Entropy loss function (1.7). The Bayes estimator $\hat{\theta}_G$ of θ under GELF (1.8) is given by

$$\hat{\theta}_G = \left[E_\theta(\theta^{-c_1})\right]^{(-1/c_1)}, \tag{1.9}$$

provided that the expectation $E_\theta(\theta^{-c_1})$ exists and is finite. Here, E_θ denote the expectation w.r.t. the posterior p.d.f. of θ.

Note that if we put $c_1=-1$ in (1.9), it provides the Bayes estimator under squared error loss function (SELF) L_1, which associates equal importance to the losses for over estimation and under estimation of equal magnitudes.

In this paper, the MLE's for the parameter θ of the EG distribution and its reliability function $R(x)$ for a specified x are derived in Section 2.1. In Section 2.2 Bayes estimators are obtained under GELF and SELF. Estimation of the parameters has been considered for a type II censored sample from p.d.f. (1.1). Finally, numerical illustrations and comparisons are presented in Sections 3 and 4 respectively.

2. CLASSICAL AND BAYESIAN ESTIMATION OF θ AND R

In a typical life test experiment, n identical objects are placed under test and exact times of failure are recorded. Usually, life tests are time consuming and costly. Therefore, at some predetermined fixed time τ or after predetermined fixed number of failures r, the test may be terminated. In both cases, the data collected consist of observations $\mathbf{t} = (t_1, t_2, ..., t_r)$ and units survived, beyond the time of termination τ in the former case and beyond the r^{th} failure t_r in the latter, remains unobserved. In a censored case, where τ is fixed and r is random, the censoring is said to be type I. On the other hand, when r is fixed and time of termination τ is random, the censoring is said to be type II. For both type I and type II censoring, Cohen [3] gave the likelihood function as

$$l(\mathbf{t}|\theta) = \frac{n!}{(n-r)!} \prod_{i=1}^{r} f(t_{(i)}|\theta) \left[1 - F(t_0)\right]^{(n-r)}, \tag{2.1}$$

where $f(t_{(i)}|\theta)$ and $F(t_0)$ are the density and distribution functions respectively. For type I censoring $t_0 = \tau$ and for type II censoring $t_0 = t_r$. Hence, expressions for the estimators of parameters under type I censoring can easily be obtained from the corresponding expressions of estimators for type II censoring just by replacing τ in place of t_r. Therefore, in the following Sections, we have considered the problem of estimation under type II censoring only.

2.1. Maximum likelihood estimators

Let us consider that n identical items whose life time follow the p.d.f. (1.1), are put on test. The test is terminated, as soon as, we observe r ordered failure times, say $t_1 < t_2 < \cdots < t_r$. Naturally, $t_1, t_2, ..., t_r$ constitute type II censored sample. Consider that the life time of the items follow distribution (1.1). Substituting $f(t|\theta)$ and $F(t)$ from (1.1) and (1.2) in (2.1), the likelihood function is obtained as

$$l(\mathbf{t}|\theta) = \frac{n!}{(n-r)!}\, \theta^r e^{-T} (1 - V^\theta)^{n-r}, \tag{2.2}$$

where

$$u_i = 1 - e^{-t_i}(t_i + 1), \quad V = 1 - e^{-t_r}(t_r + 1) \quad \text{and} \quad T = \sum_{i=1}^{r} \big(t_i - \ln t_i - (\theta - 1) \ln u_i\big) .$$

It may be verified that the MLE $\hat{\theta}_M$ of θ is the solution of the following equation

$$\text{(2.3)} \qquad \hat{\theta}_M = \frac{r}{(n-r) \ln V (V^{-\hat{\theta}_M} - 1)^{-1} - \sum_{i=1}^{r} \ln u_i} .$$

It may be noted that this is an implicit equation in $\hat{\theta}_M$, so it can not be solved analytically. We propose to solve it by using numerical iteration method, particularly Newton–Raphson method.

Using the invariance property, the MLE $\hat{R}_M$ of R may be obtained by replacing θ by its MLE $\hat{\theta}_M$ in (1.3). The same is, therefore, given by

$$\text{(2.4)} \qquad \hat{R}_M = 1 - \big[1 - e^{-t}(t+1)\big]^{\hat{\theta}_M} .$$

2.2. Bayes estimators

2.2.1. Bayes estimator of θ

For Bayesian estimation, we need to specify a prior distribution for the parameter. Consider a Gamma prior for θ having p.d.f.

$$\text{(2.5)} \qquad g(\theta) = \frac{\delta^{\nu}}{\Gamma(\nu)} e^{-\delta\theta} \theta^{\nu-1} , \qquad \theta > 0, \quad \delta > 0, \quad \nu > 0 .$$

Using Bayes theorem for combining (1.1) and (2.5), we get the posterior of θ given $\mathbf{t}$ as follows

$$\text{(2.6)} \qquad h_1(\theta|\mathbf{t}) = \frac{(\delta + q)^{\nu+r}}{k\,\Gamma(\nu + r)} e^{-(\delta+q)\theta} \theta^{\nu+r-1} \big(1 - V^{\theta}\big)^{(n-r)} ,$$

where

$$k = \sum_{j=0}^{n-r} w(j) \left(1 - \frac{jp}{\delta + q}\right)^{-(\nu+r)}, \qquad w(j) = (-1)^j \binom{n-r}{j} ,$$

$$u = \prod_{i=1}^{r} \big[1 - e^{-t_i}(t_i + 1)\big] , \qquad q = -\ln u \qquad \text{and} \qquad p = \ln V .$$

Using (1.9), the Bayes estimator of θ under GELF for the posterior (2.6) is obtained as

$$\hat{\theta}_G = \frac{1}{\delta+q}\left(\frac{\Gamma(\nu+r-c_1)}{\Gamma(\nu+r)}\right)^{-(1/c_1)}\left(\frac{k_1}{k}\right)^{-(1/c_1)}, \tag{2.7}$$

provided $\nu + r > c_1$, where

$$k_1 = \sum_{j=0}^{n-r} w(j)\left(1 - \frac{jp}{\delta+q}\right)^{-(\nu+r-c_1)}.$$

It can easily be verified that the Bayes estimator of θ under SELF for the posterior (2.6) is

$$\hat{\theta}_S = \frac{\nu+r}{\delta+q} \times \frac{k_{11}}{k}, \tag{2.8}$$

where

$$k_{11} = \sum_{j=0}^{n-r} w(j)\left(1 - \frac{jp}{\delta+q}\right)^{-(\nu+r+1)}.$$

2.2.2. Bayes estimator of R

The posterior p.d.f. of R, given $\mathbf{t}$, can be obtained from the posterior p.d.f. (2.6), using the transformation (1.3). After simplification, it reduces to

$$h_2(R|t_1,t_2,...,t_r) = \frac{Q^{\nu+r}}{\Gamma(\nu+r)k}\big(\phi_1(R)\big)^{\nu+r-1} e^{-(Q-1)\phi_1(R)}\big(1 - V^{Z\phi_1(R)}\big)^{(n-r)}, \tag{2.9}$$

where

$$\phi_1(R) = \ln(1-R)^{-1}, \quad Q = Z(\delta+q), \quad Z = 1/\ln(z^{-1}), \quad z = z(t) = 1 - e^{-t}(t+1).$$

Now, the Bayes estimator of R under GELF relative to the posterior (2.9) is obtained as

(2.10)

$$\hat{R}_G = \left[1 + \frac{1}{k}\sum_{j=0}^{n-r}\sum_{l=1}^{\infty}\frac{c_1(c_1+1)\cdots(c_1+l-1)}{l}\,\omega(j)\left(1 + \frac{l - jZp}{Q}\right)^{-(\nu+r)}\right]^{(-1/c_1)}.$$

Putting $c_1 = -1$ in (2.10), we get the Bayes estimator of R under SELF as

$$\hat{R}_S = \frac{1}{k}\sum_{j=0}^{n-r} w(j)\,(k_{12} - k_{13}), \tag{2.11}$$

where

$$k_{12} = \left(1 - \frac{j\,Z\,p}{Q}\right)^{-(\nu+r)} \quad \text{and} \quad k_{13} = \left(1 - \frac{j\,Z\,p - 1}{Q}\right)^{-(\nu+r)}.$$

It may be noted that the expression for $\hat{R}_S$ obtained above is the same as that obtained by Shawky and Bakoban [9].

3. COMPARISON OF ESTIMATORS

In this Section, we shall compare the estimators obtained under GELF with the corresponding Bayes estimators under SELF and the MLE. The estimators $\hat{\theta}_M$ and $\hat{R}_M$ denote the MLE's of the parameter θ and the reliability function $R(x)$ for a specified x respectively. $(\hat{\theta}_G,\, \hat{R}_G)$ and $(\hat{\theta}_S,\, \hat{R}_S)$ are the corresponding Bayes estimators under GELF and SELF. The comparisons are based on the **risks(average loss over sample space)** of the estimators of the parameters θ and R of the considered model. The exact expressions for the risks can not be obtained, therefore, the risks of the estimators are estimated on the basis of Monte-Carlo simulation study of 5000 samples. It may be noted that the risks of the estimators under type II censoring will be a function of sample size n, number of observations r, parameters δ and ν of prior distribution, parameter θ of the model, x and loss function parameter c_1. In order to consider a variation of these values, we have obtained the simulated risks for $n = 15\,[5]\,25$ and $r = 8\,[2]\,14$. The various values of the hyper parameters considered here are $\delta = 1\,[1]\,7$ and $\nu = 1\,[1]\,7$. We vary $c_1 = -3.0\,[0.5]\,3.0$. θ and x are arbitrarily taken as 1.5 and 0.5 respectively. After an extensive study of results, conclusions are drawn regarding the behavior of the estimators. It may be mentioned here that because of space restriction, results for all the variations in the parameters are not shown here. Only selected figures are included. In the figures $R_G(\cdot)$ and $R_S(\cdot)$ denote the risks of $(\cdot)$ under GELF and SELF respectively.

Firstly, we observed the impact of variation of sample size n and number of observations r under type II censoring on the risks of estimators $\hat{\theta}_G$, $\hat{\theta}_S$, $\hat{\theta}_M$, $\hat{R}_G$, $\hat{R}_S$ and $\hat{R}_M$, keeping the value of other parameters fixed. It is observed that as n increases, the risks of all the estimators decrease in all the considered cases; although the decrease is more for $\hat{\theta}_M$ and $\hat{R}_M$. For large sample sizes, the difference between the risks of the estimators are negligibly small. It is further observed that if we increase the value of r keeping the sample size n fixed, there is a slight decrease in the risks of the estimators (to save the space corresponding figures are not included in this paper). Keeping these points in mind, we have presented the figures with (n, r) equal to $(15, 12)$ only.

Let us now study the effect of variation of loss parameter c_1 on the risks of the estimators. It is re-iterated that the positive sign of the loss parameter c_1

indicates that over estimation is more serious than under estimation and the magnitude of c_1 indicates its intensity. It is observed that, in general, the risks of the estimators under GELF increases, as c_1 increases (see figure 1). The increase in the risks is more for $\hat{\theta}_M$ as compared to the other estimators. For almost all values of c_1, the risk of $\hat{\theta}_G$ under GELF is found to be least among the considered estimators. It is interesting to remark here that $\hat{\theta}_G$ has the least risk under SELF also. It is further noted that for reliability estimation, $\hat{R}_M$ has the smallest risk under GELF (see figure 3). For negative values of c_1, the behavior of risks of estimators under GELF is more or less similar to the one obtained for positive c_1 (see figure 2).

While studying the effect of variation in the value of ν, we observed that, in general, under both loss functions, the risks of the estimators of θ (except for $\hat{\theta}_M$) increase as ν increases. It is also seen that $\hat{\theta}_G$ has smaller risk compared to the risks of other estimators when $\nu \leq 4$; otherwise $\hat{\theta}_M$ has smaller risk (see figure 4). The behavior of risks of the estimators of reliability are just reverse to those for the estimators of θ. It decrease as ν increases except for $\hat{R}_M$. The smallest risk is observed for $\hat{R}_S$ as compared to the risks of others (under both the losses; namely GELF and SELF), except when $\nu \leq 2$ for which $\hat{R}_M$ has smaller risk (see figure 7). For negative values of c_1, the trend of risks as ν increases, is similar to that of positive c_1. Under GELF, the risk of $\hat{\theta}_M$ is found to be smaller than the risks of other estimators, when $\nu \geq 5$ and for $2 \leq \nu < 5$, $\hat{\theta}_S$ has smaller risk than others; but for $\nu = 1$, $\hat{\theta}_G$ has smaller risk. Under SELF, the risk of $\hat{\theta}_M$ is smaller than the risk of other estimators for $\nu \geq 3$ and for $\nu < 3$, $\hat{\theta}_S$ has smaller risk. The trend of risks of the estimators of reliability is just reverse to those of the estimators of θ; i.e., the risks, in general, decrease as ν increases. Under both the loss functions, $\hat{R}_G$ has a smaller risk than others for $\nu \geq 3$ and for $\nu < 3$, under both the loss functions, $\hat{R}_M$ has the smallest risk (see figure 6).

While observing the effect of variation in the value of δ, it is noted that for positive values of c_1, as δ increases, risks of estimators increase, in general, for fixed values of other parameters. $\hat{\theta}_M$ has smaller risk than the Bayes estimators $\hat{\theta}_G$ and $\hat{\theta}_S$ for large value of $\delta \geq 6$, while for $2 \leq \delta < 6$, $\hat{\theta}_S$ has the smallest risk, but for $\delta = 1$, $\hat{\theta}_G$ has smallest risk. The trend remains more or less the same under both loss functions (see figure 5), and in case of estimators of reliability, it is observed that the risk of the MLE, $\hat{R}_M$, is smaller than those of Bayes estimators $\hat{R}_G$ and $\hat{R}_S$ (see figure 9). For negative values of c_1, it is observed that as δ increases, risks increase, in general, except for the MLE's. This trend is similar to that for positive c_1. However, for $\delta \leq 3$, $\hat{\theta}_G$ has smaller risk under GELF and for rest of the values of δ, $\hat{\theta}_M$ performs better than the other estimators. For $3 \leq \delta \leq 6$, $\hat{\theta}_G$ performs better than others under SELF (see figure 8).

It is worthwhile to mention here that the risks of the estimators under type I censored data were also obtained for $\theta = 1.5$ and $\tau = 3$, taking values of $c_1 = -3\,[0.5]\,3$, $n = 15\,[5]\,25$, $\delta = 1\,[1]\,7$ and $\nu = 1\,[1]\,7$. After an extensive study of the results, thus obtained, we observed that the risks of the estimators under type I censored data behave similarly to the risks of the estimators under type II censored data with little changes in the magnitude of the risks. Thus, we may infer that censoring mechanism has no significantly different effect on the performance of the proposed estimators so far as behavior of their risks are concerned.

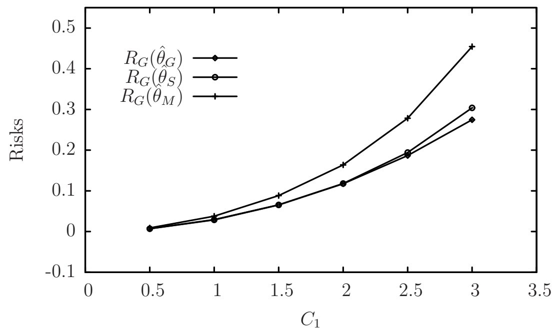

Figure 1: Risks of estimators of θ under GELF for fixed $n = 15$, $r = 12$, $\theta = 1.5$, $\delta = 1$, $\nu = 1$, for positive values of c_1.

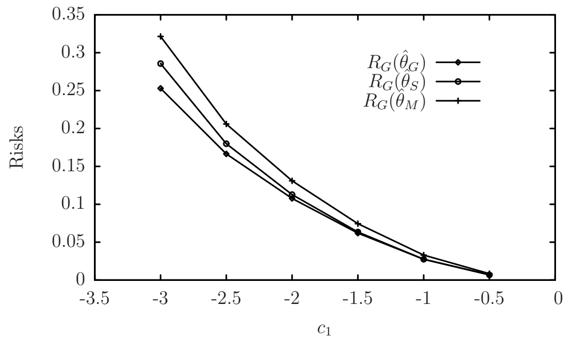

Figure 2: Risks of estimators of θ under GELF for fixed $n = 15$, $r = 12$, $\theta = 1.5$, $\delta = 1$, $\nu = 1$, for negative values of c_1.

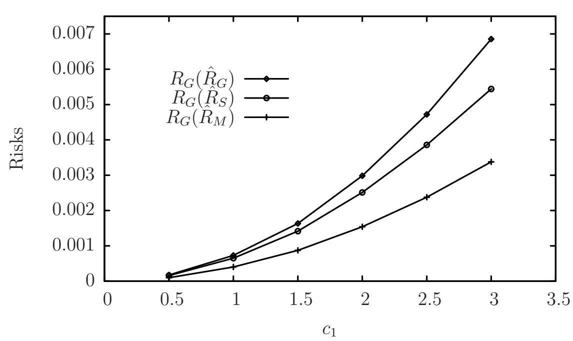

Figure 3: Risks of estimators of R under GELF for fixed $n = 15$, $r = 12$, $x = 0.5$, $\theta = 1.5$, $\delta = 1$, $\nu = 1$.

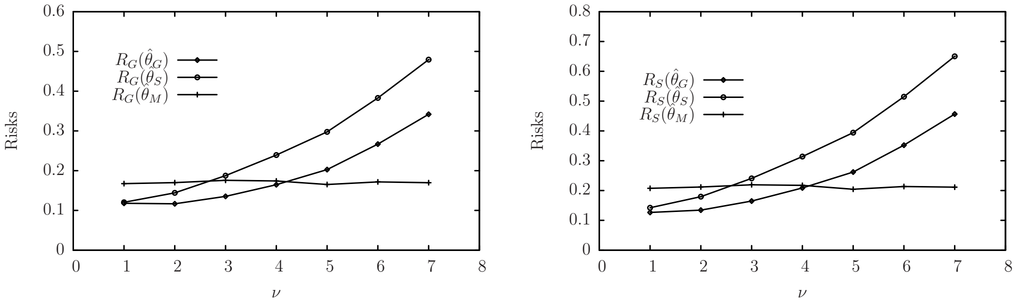

Figure 4: Risks of estimators of θ under GELF (left) and SELF (right) for fixed $n = 15$, $r = 12$, $\theta = 1.5$, $\delta = 1$, $c_1 = +2.0$.

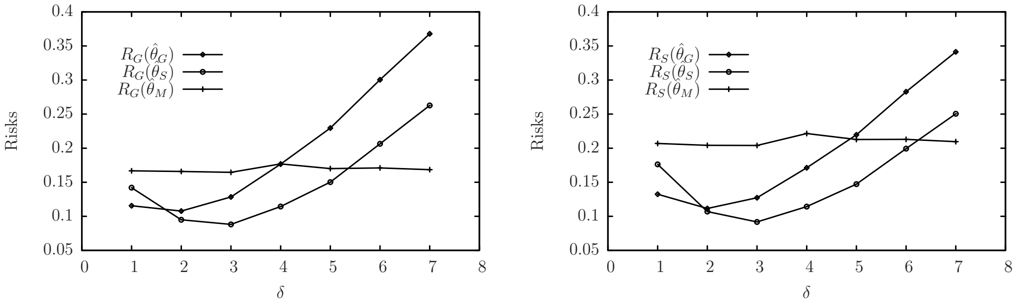

Figure 5: Risks of estimators of θ under GELF (left) and SELF (right) for fixed $n = 15$, $r = 12$, $\theta = 1.5$, $\nu = 2$, $c_1 = +2.0$.

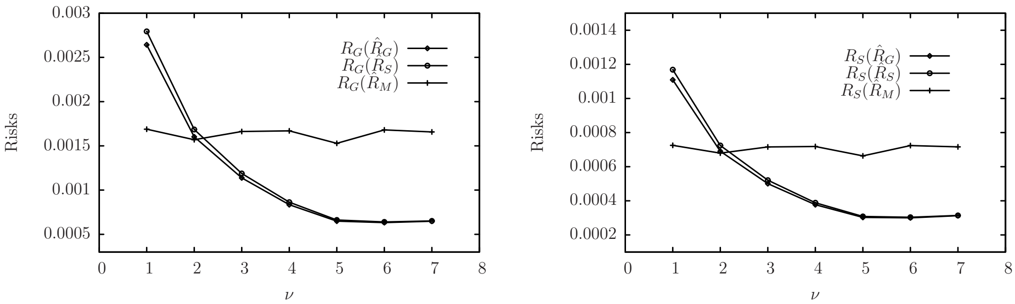

Figure 6: Risks of estimators of R under GELF (left) and SELF (right) for fixed $n = 15$, $r = 12$, $t = 0.5$, $\theta = 1.5$, $\delta = 1$, $c_1 = -2.0$.

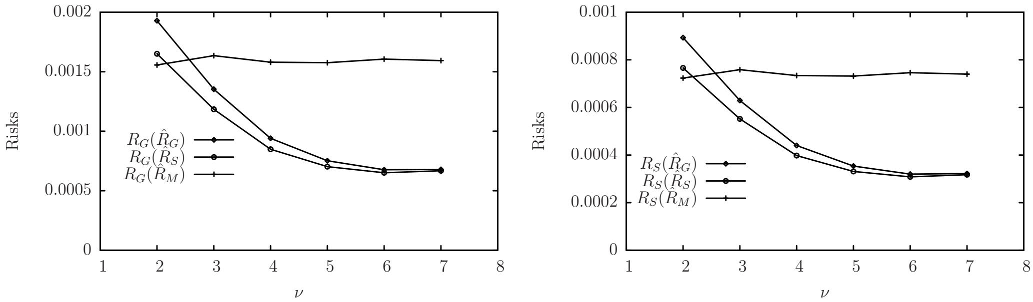

Figure 7: Risks of estimators of R under GELF (left) and SELF (right) for fixed $n = 15$, $r = 12$, $x = 0.5$, $\theta = 1.5$, $\delta = 1$, $c_1 = 2.0$.

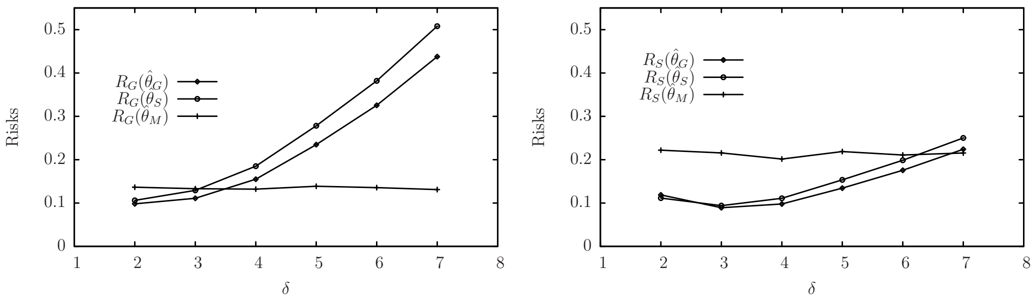

Figure 8: Risks of estimators of θ under GELF (left) and SELF (right) for fixed $n = 15$, $r = 12$, $\theta = 1.5$, $\nu = 2$, $c_1 = -2.0$.

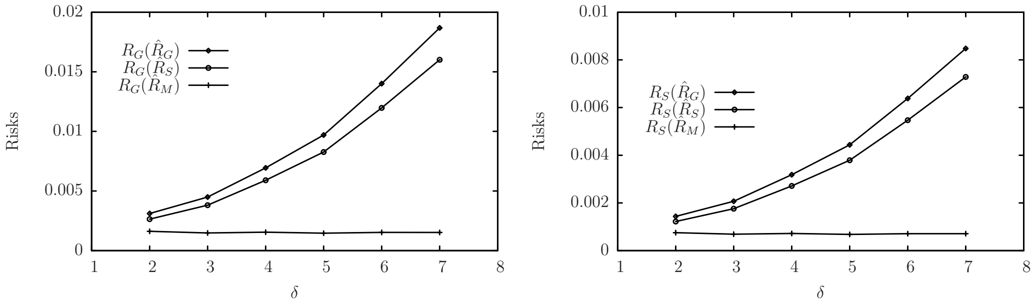

Figure 9: Risks of estimators of R under GELF (left) SELF (right) for fixed $n = 15$, $r = 12$, $x = 0.5$, $\theta = 1.5$, $\nu = 2$, $c_1 = +2.0$.

4. CONCLUSION

On the basis of the discussion given in the previous Section, we may conclude that the proposed estimator $\hat{\theta}_G$ performs better than $\hat{\theta}_S$ and $\hat{\theta}_M$ for small values of δ and ν and $c_1 \leq -1.0$ (when under estimation is more serious than over estimation) in the sense of having smaller risk. Contrary to it, when over estimation is more serious than under estimation, our proposed estimator performs well when $\delta = 1$, $\nu \leq 4$ and $c_1 \geq 2$. Thus, the use of the proposed estimator $\hat{\theta}_G$ is recommended even under quadratic loss function. In case of estimation of reliability function, our proposed estimator $\hat{R}_G$ performs better than $\hat{R}_S$ and $\hat{R}_M$ when $c_1 = -2$, $\delta = 1$ and $\nu \geq 3$. In other cases, $\hat{R}_G$ has slightly higher risk than $\hat{R}_S$ and $\hat{R}_M$. Therefore, the proposed estimator $\hat{R}_G$ is recommended for use only if under estimation is more serious and hyper parameter ν is large.

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