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NON-STATIONARY MODELLING OF EXTREME TEMPERATURES IN A MOUNTAINOUS AREA OF GREECE

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Abstract:

- The generalised extreme value (GEV) distribution is often fitted to environmental time series of extreme values such as annual maxima and minima of temperatures. It is often necessary to allow the distribution's parameters to depend on time or other covariates (non-stationary GEV). Increasingly, model fitting within the GAMLSS framework is being used as an alternative approach. A case study is presented of temperature extremes in a mountainous area of Greece divided into nine zones by altitude. Model fitting supported non-stationary GEV models for temperature with the location parameter depending linearly on year and zone, showing the expected dependence on altitude along with an increasing trend in annual maxima and declining trend in annual minima. The scale parameter for maxima depended on zone, with greater variability at higher altitudes. The scale parameter for minima increased over time. Fitting non-stationary Inverse Gaussian, Lognormal and Gamma distributions within the GAMLSS framework identified the same dependence on zone and year. There was little difference in goodness of fit of the various distributions.

Key-Words:

- *modelling extremes; GEV distribution; GAMLSS; non-stationary models; extreme temperatures.*

AMS Subject Classification:

- 60G70, 62P12.

1. INTRODUCTION

The study of extreme values in climatological time series is an area of intense scientific activity. Examples of this type of data are series of annual or monthly maxima of precipitation or temperature. This is the block maximum approach to defining the extremes of time series; the alternative peaks-over-threshold approach will not be considered in this paper (Beirlant *et al.*, [1]; Chavez-Demoulin and Davison, [2]; Gomes and Guillou, [9]; Scarrott and Macdonald, [22]). An early method of analysis that subsequently became well-established was to fit the Generalised Extreme Value distribution, assuming a series of independently and identically distributed values over time (Jenkinson, [11]). However, it has become increasingly clear that many series do not possess this property of stationarity, because of natural climate variability or anthropogenic climate change (Jain and Lall, [10]; Milly *et al.*, [18]; Serinaldi and Kilsby, [23]). Consequently, it becomes necessary to move from stationary to non-stationary models.

Introducing non-stationarity within the framework of standard statistical distributions requires extended models with covariate-dependent changes in one or more of a distribution's parameters (Coles, [4]). For example, a trend towards higher temperatures could be represented by the time-dependence of the parameter that represents the distribution's mean, or increased variability in a rainfall series by time-dependence of the parameter that is associated with the distribution's variance. Spatial trends and dependence on any other available covariates can be represented in a similar way.

2. STATISTICAL MODELLING

2.1. Generalised extreme value (GEV) distribution

The GEV distribution is widely employed in the environmental sciences and elsewhere for modelling extremes (Reiss and Thomas, [20]). It depends on three parameters: location μ, scale σ and shape ξ. In the non-stationary GEV distribution (El Adlouni *et al.*, [7]; Leclerc and Ouarda, [14]), these parameters are expressed as a function of time t and possibly other covariates (Coles, [4]). If, as is usually done, we allow non-stationarity of the location and scale parameters but not of the shape parameter, this non-stationary GEV($\mu(t)$, $\sigma(t)$, ξ) distribution has distribution function

$$F\big(y;\mu(t),\sigma(t),\xi\big) \;=\; \exp\left\{-\left[1+\xi\,\frac{y-\mu(t)}{\sigma(t)}\right]^{-1/\xi}\right\}. \tag{2.1}$$

In the simplest case, the following regression structures could be considered for the location and scale parameters

$$\begin{aligned} \mu(t) &= \mu_0 + \mu_1 t + \mu_2 t^2 + \mu_3 t^3 , \\ \sigma(t) &= \exp\bigl(\sigma_0 + \sigma_1 t + \sigma_2 t^2 + \sigma_3 t^3\bigr) , \end{aligned} \tag{2.2}$$

allowing up to cubic dependence on time t. We denote by GEVjk the model with time dependence of order j in the location parameter and order k in the scale parameter. A convenient tool for fitting either stationary or non-stationary GEV distributions is the *gev.fit* function in the R package 'ismev' (available from `http://cran.r-project.org/package=ismev`), which employs the maximum likelihood method. Other relevant R packages are listed by Gomes and Guillou ([9]). Bayesian and other estimation methods are discussed by, for example, Beirlant *et al.* ([1]), Chavez-Demoulin and Davison ([2]), and Gomes and Guillou ([9]). The estimation of the shape parameter ξ may sometimes cause difficulty, as observed by Coles and Dixon ([5]) who proposed using a penalized likelihood function to avoid this problem. Similarly, Martins and Stedinger ([17]) proposed restricting the estimate of ξ to fall within the range $[-0.5, +0.5]$ by using a suitable prior distribution. However, we have not encountered any difficulty in the estimation of ξ in the practical problems that we have investigated.

2.2. GAMLSS

Generalised additive models for location, scale and shape (GAMLSS; Rigby and Stasinopoulos, [21]) represent a very wide class of non-stationary distributions. GAMLSS provide a highly flexible framework for modelling, because as many as four parameters of a distribution chosen from an extensive family are allowed to depend on covariates. The first applications of GAMLSS to meteorological data appear to have been by Villarini and colleagues, who examined the fit of Gumbel, Weibull, Gamma, Lognormal and Logistic distributions to data on rainfall and temperature in Rome (Villarini *et al.*, [25]), and the first four of these to flood peaks in the United States (Villarini *et al.*, [26]). Further examples of its application are now quite common; recent examples include Lopez and Frances ([15]), who fitted the Gumbel, Lognormal, Weibull, Gamma and Generalized Gamma distributions, Garcia Galiano *et al.* ([8]) (fitting the Lognormal, Weibull and Gamma distributions) and Machado *et al.* ([16]) (Lognormal, GEV and two-component extreme value distributions).

The general format of the model for parameter θ_k is

$$g_k(\theta_k) = X_k \beta_k + \sum_{j=1}^{J_k} Z_{jk}\, \gamma_{jk} , \tag{2.3}$$

where g_k is a link function, X_k is a design matrix containing the values of J_k covariates for each of n independent observations, β_k is a parameter vector of length J_k, Z_{jk} is another known design matrix of dimension $n \times q_{jk}$ and γ_{jk} is a q_{jk}-dimensional random vector. In the absence of random effects, the first term on the left-hand side of (2.3) gives a parametric linear model; in this case, the advantages of GAMLSS over generalised linear models or generalised additive models are its not being restricted to exponential family distributions and its ability to model several parameters of the distribution, not just the mean. Furthermore, Rigby and Stasinopoulos ([21]) and Stasinopoulos and Rigby ([24]) demonstrate how the second term of (2.3) can be used to construct a wide variety of models, although this generality will not be required in the present paper.

GAMLSS modelling is implemented in the R package 'gamlss' (`http://cran.r-project.org/package=gamlss`; Stasinopoulos and Rigby, [24]), which makes it easy to include features such as random effects or non-polynomial dependence on covariates by means of splines. The method of fitting is penalized maximum likelihood. A recent extension to 'gamlss.spatial' (`http://cran.r-project.org/package=gamlss.spatial`) offers a facility for spatial modelling by including Markov Random Field additive terms.

2.3. Model selection

When searching for the best fitting model among many alternatives, it is important to have objective procedures for making the selection from the various candidates. The likelihood ratio test can be used if the models are hierarchically nested. The Akaike information criterion (AIC) and the Bayesian information criterion (BIC) are also widely employed for model selection. If $\hat{\ell}$ is the maximized value of the likelihood from a model that contains p parameters, and n is the sample size, these criteria are defined as

$$AIC_C = -2\hat{\ell} + 2p + \frac{2p(p+1)}{n-p-1} \tag{2.4}$$

(this is the corrected AIC — the third term is a small-sample adjustment) and

$$BIC = -2\hat{\ell} + p \ln n \,. \tag{2.5}$$

The preferred model minimizes the chosen criterion although alternative models with values close to the minimum should not be ignored. More details of model selection procedures can be found in Claeskens and Hjort ([3]), for example.

Panagoulia *et al.* ([19]) carried out a simulation study in order to evaluate empirically the performance of the AICc and BIC in identifying the true model among the set of models GEVjk ($j = 0, 1, 2, 3$; $k = 0, 1, 2, 3$), for samples of sizes $n = 20, 50$ or 100. Both criteria had high success rates in detecting non-stationarity.

The BIC was the more successful in identifying the correct model: over 80% of the time for $n = 50$ and over 90% for $n = 100$, although these percentages obviously depend on the parameter values selected for the study. AICc was better for $n = 20$, although this is a small sample in relation to the number of parameters in some of these models and neither selection criterion performed very well.

2.4. Uncertainty

Apart from obtaining a description of the phenomenon, one of the major objectives of fitting a distribution to climate data is to obtain estimates of its quantiles, especially those related to the return periods of extreme events: for example, the upper 1% point of the distribution of annual maxima corresponds to a $1/.01 = 100$-year return period. Good estimation of the uncertainty in extreme levels can be as important as the estimate of the level itself (Coles, [4]; Khaliq *et al.*, [12]). Parametric confidence intervals based on a normal distribution approximation cannot be expected to be accurate for extreme quantiles; that is, their actual coverage probabilities will not be close to the nominal values. As a result, confidence interval construction by bootstrap methods has been examined for GEV models, first by Kysely ([13]) in the stationary case and subsequently by Panagoulia *et al.* ([19]) in the non-stationary case. Amongst several methods compared, the best was found to be the parametric bootstrap with confidence intervals constructed by the bias corrected and accelerated (BCa) technique. Serinaldi and Kilsby ([23]) expressed a preference for percentile parametric bootstrap confidence intervals, although this appears to be based on general considerations rather than detailed studies. However, they warned that the estimation of extreme quantiles is inherently so uncertain that the discrepancy between different types of confidence intervals is not of major relevance.

Uncertainty in predictions also stems from model selection. The above confidence intervals are based on the assumption that the correct model has been selected and take no account of the alternatives that were considered. Model averaging procedures exist and are used in many contexts to overcome this objection, especially in the Bayesian framework, but will not be considered here.

3. CASE STUDY

3.1. Data

Our analysis concerns time series of meteorological data from one catchment area in the mountains of Central Greece. Further description of this location can be found in Panagoulia *et al.* ([19]), where analyses of annual maxima of rainfall over the whole catchment area are carried out for historical data and for data simulated under climate change scenaria. In the present paper, analyses are carried out for annual maxima and minima of temperature over the period 1972–1992, over the whole area and in nine zones corresponding to a partition of the area by elevation.

3.2. GEV modelling

The series of minima can be analysed by GEV modelling after taking the negative of its values and fitting the same models as to the series of maxima (Chavez-Demoulin and Davison, [2]). The series of annual extremes for the entire area do not appear to be stationary, as the GEV*10* model offers significantly improved fit over the GEV*00* model (comparing minus twice the change in log-likelihood to the chi-squared distribution with one degree of freedom, $p = 0.05$ for maximum temperatures, $p = 0.01$ for minima). The smooth curves fitted to the annual minima in Figure 1 and annual maxima in Figure 2 demonstrate a decreasing and an increasing trend, respectively. The suggestion in Figure 1 of greater variance of the minima in the later years is not borne out by statistical

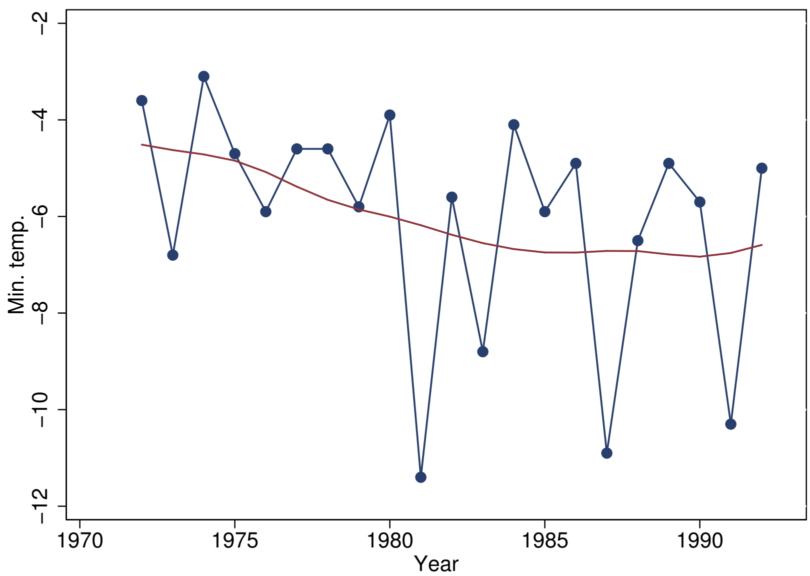

Figure 1: Annual minimum temperatures in °C over the whole study area, with trend fitted by locally weighted scatterplot smoothing.

tests for a linear trend in the scale parameter ($p = 0.40$ for GEV*11* versus GEV*10*; $p = 0.91$ for the corresponding test for the maxima). In contrast to the results of the analysis of rainfall data in Panagoulia *et al.* ([19]), the GEV model for temperatures did not reduce to the Gumbel ($\xi = 0$) as the former had much better fit than the latter (AIC 96.8 compared to 102.1).

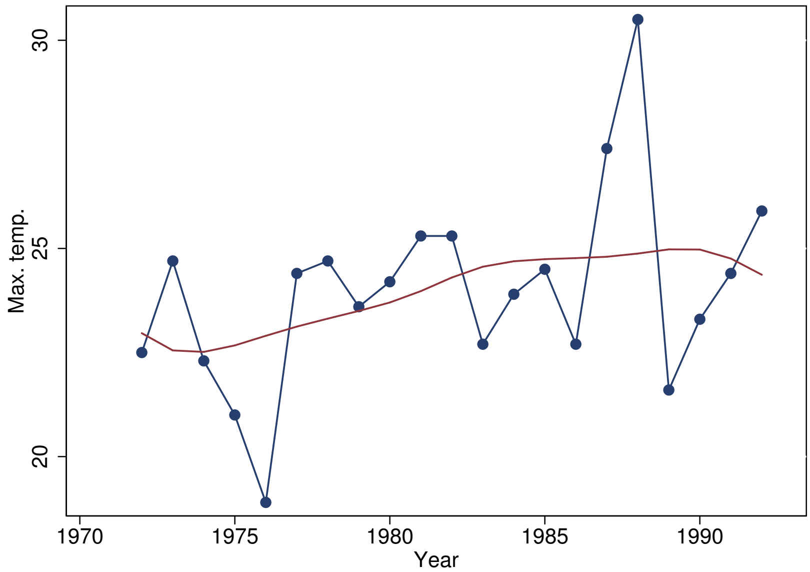

Figure 2: Annual maximum temperatures in °C over the whole study area, with trend fitted by locally weighted scatterplot smoothing.

Parameter estimates obtained from fitting the stationary GEV*00* model to the data from each zone separately are shown in Table 1. There appear to be trends with zone, that is, with altitude. In particular, as would be expected,

Table 1: Fitting stationary GEV to annual maximum and minimum temperatures in each zone separately: estimates of location μ, scale σ and shape ξ.

Zone	Maxima			Minima		
	μ	σ	ξ	μ	σ	ξ
1	25.2	2.05	−0.23	2.16	1.96	−0.310
2	24.4	2.05	−0.19	2.63	1.91	−0.230
3	23.9	2.08	−0.18	3.15	1.78	−0.080
4	23.5	2.12	−0.17	3.97	1.58	0.080
5	23.0	2.22	−0.17	5.07	1.43	0.210
6	22.8	2.49	−0.19	6.24	1.56	0.170
7	22.5	2.73	−0.11	7.70	1.85	0.090
8	22.3	2.85	−0.02	8.76	2.05	0.040
9	22.0	3.06	0.06	10.24	2.28	0.002
Standard error:	0.53	0.35	0.11	0.46	0.34	0.18
median (range)	(0.48–0.75)	(0.32–0.56)	(0.10–0.16)	(0.36–0.57)	(0.29–0.41)	(0.17–0.20)

the location parameter appears to decline as the altitude increases from Zone 1 to Zone 9 for the maxima and also — apparently to a much greater degree — for minus the minima. Furthermore, the relationship with zone is very close to linear. Also, fitting the GEV*10* model separately in each zone (results not shown) suggests time dependence of the scale in many zones, as was noted above in the analysis of the entire area. The next step was to fit models to the annual maxima and minima by year and zone, allowing various forms of dependence on both of these covariates.

The best fitting model for annual maxima, selected using likelihood ratio tests and the AICc and BIC criteria, included linear dependence of μ on zone and year, and log-linear dependence of σ on zone. The shape parameter ξ did not depend on either covariate, despite the indication of a trend in Table 1 which may have been due to correlations between the estimates of the three parameters. The fitted model was:

$$\begin{aligned}
\hat{\mu} &= \underset{(0.36)}{25.00} - \underset{(0.073)}{0.285}\,Zone + \underset{(0.030)}{0.118}\,(Year - 1982)\,, \\
\ln\hat{\sigma} &= \underset{(0.109)}{0.554} + \underset{(0.021)}{0.068}\,Zone\,, \\
\hat{\xi} &= \underset{(0.045)}{-0.142}\,.
\end{aligned}$$

(Standard errors are shown in parentheses below the parameter estimates to which they refer.) We note that the estimate of ξ is clearly significantly different from zero, meaning that the Gumbel distribution is not suitable here. This is different from the finding for rainfall over the same catchment area in Panagoulia *et al.* ([19]), although that analysis was for the total area not broken down by zone. The corresponding analysis for (minus) the annual minima, produced a slightly different model, with σ depending on year instead of zone. The fitted model was:

$$\begin{aligned}
\hat{\mu} &= \underset{(0.314)}{0.438} + \underset{(0.060)}{1.043}\,Zone + \underset{(0.024)}{0.121}\,(Year - 1982)\,, \\
\ln\hat{\sigma} &= \underset{(0.063)}{0.579} + \underset{(0.011)}{0.026}\,(Year - 1982)\,, \\
\hat{\xi} &= \underset{(0.063)}{0.004}\,.
\end{aligned}$$

In this case, the estimate of ξ is clearly not significantly different from zero, implying that the Gumbel distribution could be employed. Comparing the two sets of equations, it is noticeable that annual maxima are increasing and annual minima are decreasing, meaning that the temperature range is increasing. In fact, the coefficients representing the time dependence of the location μ are almost equal: annual maxima are increasing at the same rate as annual minima are decreasing. However, the coefficient of the dependence of μ on altitude is much bigger for minima than for maxima. This gives an expected result, that minimum temperatures fall more steeply than maximum temperatures with altitude. The results for scale show increasing variability of minima with time — which is what Figure 1 indicated, but did not emerge from the analysis for the entire area aggregated across zones. The variability of maxima increases at higher altitude but is not changing with time.

3.3. GAMLSS modelling

There is no theory to guide the choice of which distribution to fit from among the many available in GAMLSS. We carried out the modelling using the Inverse Gaussian, Gamma and Lognormal distributions, allowing non-stationarity in the form of polynomial dependence of the parameters on year and zone just as we did for the GEV distribution. The preferred models coincided with those chosen for the GEV. We found that the fits of these non-stationary distributions were almost identical. For maxima, AIC values were 911.2 for the Inverse Gaussian distribution, 910.6 for the Lognormal and 912.0 for the Gamma distribution. Graphs demonstrating goodness-of-fit are not presented because the lines showing each distribution are virtually indistinguishable. Furthermore, estimated percentiles were very close.

Close similarity of fits between different models is probably a usual feature of modelling data of this kind. For example, Villarini *et al.* ([26]) analysed annual flood peaks from many stations using GAMLSS and found (see their Table 7) that the Lognormal distribution provided the best fit in 16 sets of data, the Gamma in seven, the Gumbel in 5 and the Weibull in one. In the absence of theory to guide the choice, the preference for one or the other may well just be a matter of sampling variability.

4. CONCLUSION AND COMMENTS

When the underlying distribution is stationary, the choice of the GEV distribution for modelling extremes is well supported on theoretical grounds, precisely because it is an extreme value distribution. That is, it is a form that necessarily arises in the limit to describe the distribution of the maxima of a series of independent and identically distributed random variables (Cox *et al.*, [6]; Gomes and Guillou, [9]). In the non-stationary case, however, the original sequence does not consist of identically distributed variables. We are not aware that any limiting form necessarily arises in this case. This suggests that there is no compelling reason to use the non-stationary GEV in preference to many other distributions that are available. The choice of distribution then becomes entirely empirical. This is the approach that seems to have been taken in the various papers that have appeared in the literature so far on the application of GAMLSS to meteorological and related data. These papers tend to demonstrate the possibilities that this flexible approach to modelling offers but not to go on to draw conclusions about which models are the most appropriate on general grounds. Searching through alternative distributions — which the GAMLSS framework tends to encourage — also adds an extra layer of uncertainty to the model selection procedure which ought to be accounted for in predictions.

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